A Direttissimo Algorithm for Equidimensional Decomposition

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We describe a recursive algorithm that decomposes an algebraic set into locally closed equidimensional sets, i.e. sets which each have irreducible components of the same dimension. At the core of this algorithm we put our own spin on a classical incremental solving strategy and work with Gröbner bases to encode locally closed equidimensional sets. Equipped with this, our algorithm avoids generic projections and other genericity assumptions on the input frequently made when decomposing polynomial systems such as Noether position. Built around the philosophy to split a given polynomial system as often as possible, as irredundantly as possible, our algorithm is both simple to describe and implement and produces fine decompositions on more structured systems where ensuring genericity assumptions often destroys the structure of the system at hand. The algorithm has been implemented using the computer algebra system OSCAR with the additional enhancement of a certain probabilistic data structure. Experimental results indicate that it is able to tackle some polynomial systems which are out of reach of other algorithms for equidimensional decomposition available in state-of-the-art computer algebra systems. This is joint work with Christian Eder, Pierre Lairez and Mohab Safey El Din.