GLOBAL MINIMIZATION OF ANALYTIC FUNCTIONS

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ABSTRACT. We introduce a new algorithm for minimizing a nonconvex analytic function f over $\mathcal{C}_n$, the n-dimensional hypercube $[-1,1]^n$, with probabilistic guarantees of optimality. Our work combines methods from approximation and learning theory with Gröbner basis algorithms for solving multivariate polynomial systems. Our algorithm is in essence an effective application of the Stone-Weierstrass theorem, coupled with a method for computing all critical points of a polynomial. The algorithm is as follows: we first approximate f by a polynomial p on a dense subset of sample points $\mathcal{S}$ in $\mathcal{C}_n$ using discrete least squares. We construct the polynomial approximant such that it satisfies a given error bound $\|p - f\| \leq \epsilon$ with probability larger or equal to a set $\mu \in (0,1)$. We rely on polynomial approximation error bounds in expectation and probability to guarantee the convergence of p towards f while maintaining the degree of p as low as possible. We then compute all the critical points of p through state of the art symbolic computation methods. Finally, we initiate local minimization methods at each of the critical points computed in the previous step in order to find the global minima of f over $\mathcal{C}_n$.

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