Computer algebra methods for polynomial system solving at the service of Image-Based Visual Servoing

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Abstract

Image-Based visual servoing (IBVS) refers to a class of Robotics controllers based on computer vision data, where a set of features available from a camera image is used to control the motion of a robot.

When performing Image-Based Visual Servoing, it is a known issue that using a redundant set of visual features (i.e. more than the number of degrees of freedom of the system) may lead to the appearance of local minima, that is, stable camera configurations for which the objective error function is non-zero. The convergence of the camera towards one of these equilibrium points may result in failure of the robotic task. The determination of these local minima is a challenging problem due to the complexity of the systems of polynomial equations involved. Further, correctly identifying all the local minima is a necessary step for an analysis of the global stability and convergence properties of IBVS controllers.

In this work we address the problem of computing the points of equilibrium of Image-Based visual servoing from N reference points using exact, certified computational methods. The visual features in this case are the 2D coordinates of the projection of the N points on the camera image. Then, the points of equilibrium are the critical points of a potential function representing the error norm. We model the problem as a system of polynomial equations arising from the gradient of the error potential, in the variables representing the projected coordinates of the points and their depth along the focal axis of the camera. Using `msolve`, a polynomial-based system solver, we solved this system for N = 4 reference points. However, due to the complexity of the equations, the computing times are exceedingly long (over several weeks with multi-threading computations over 12 cores). Since the local minima must be recomputed for every configuration of the reference points and for every desired end pose of the camera, we find that this formulation is insufficiently effective.

We then present an improved modeling of the problem, by defining a change of variables that exploits the symmetries of the solution set of the original system, and then performing algebraic elimination by means of Gröbner bases, leading to a new polynomial ideal with lower degree. Additionally, we also find that, in the case of planar markers (i.e. the reference points lying on the same plane), we can further reduce the degree of the polynomial ideal by imposing the coplanarity condition in the space of the state variables. With this reformulation, we can compute the critical points in the case of 4 generic reference points in a matter of 2-3 days, or in a few hours for the case of planar objects.

This work illustrates the prolific interaction between the fields of Robotics and Computer algebra, and an application of symbolic computational methods for solving systems of polynomial equations to real engineering problems.