Welfare Structure in Two-sided Random Matching Markets

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Joint work with Itai Ashlagi (Stanford) and Mark Braverman (Princeton)

From Matchings to Markets
CIRM, December 2023

Two-sided matching markets

- 8qqe_H9§_nj I nje_jO L9§_jXJ H\nne H\n_HOJ }O _LOj§ h9§H_jXJ O§Hu

Two-sided matching markets

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- **~§9@_e_§**´I }O _ §9jHO §n _jXeO]q9}§´ LO®_9§nj

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- mT§0j §«L´ **}9jLnh q}0T0}0jH0** I 09 _0} §n X0j0}9e_¶0¡ ōx«9e_§´Ÿ ® ōe«HcŸ

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Stable matching markets with logit-based preferences

- *n* 9X0j§ nj 09H\ _L0 rō°nh0jŸ 9jL ōh0jŸs °_§\ **enX_§]@9 0L** }9jLnh q}0T0}0jH0

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  Woman i's preference is generated from a logit model with stochastic vector $\hat{a}_i = (\hat{a}_{ij})_{j \in [n]} \in \mathbb{R}^n_+$

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- **~§9@_e_§´I** jn @enHc_jX q9_}
- **|9jc** nT 9j 9X0j§I _jL0² nT \0} h9§H\0L q9}§j0} _j \0} q}0T0}0jH0

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Stable matching markets with logit-based preferences

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  - Homogeneous case (uniform preferences): $\hat{A} = \hat{B} = \frac{1}{n} 1_n$

Stable matching markets with logit-based preferences

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- ¯n}c_jX 0²9hqe0 I
  - Homogeneous case (uniform preferences): $\hat{A} = \hat{B} = \frac{1}{n} 1_n$
  - Community structures: $A = B = \frac{2}{n} \begin{matrix} 1_{n=2} & 0_{n=2} \\ 0_{n=2} & 1_{n=2} \end{matrix}$

Stable matching markets with logit-based preferences

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    - Homogeneous case (uniform preferences): $\hat{A} = \hat{B} = \frac{1}{n} \mathbf{1}_n$
    - Community structures: $A = B = \frac{2}{n} \begin{pmatrix} \mathbf{1}_{n/2} & \mathbf{0}_{n/2} \\ \mathbf{0}_{n/2} & \mathbf{1}_{n/2} \end{pmatrix}$
    - Public popularities: $\hat{a}_i = \mathbf{a}$ for all i, $\hat{b}_j = \mathbf{b}$ for all j

Our objectives

ª jLO} §9jL §\O °OeT9}O §}«H§«}O _j §_ XOjO}9e hnLOeI

Our objectives

- jLO} §9jL §\O °OeT9}O §}«H§«}O _j §_ XOjO}9e hnLOeI
  - _H\ 9XOj§ 9}O hn}O **Hnhq0§_§_®O¤qnq«e9}**y
  - §\O h9}cO§ **°Oee HnjjOH§OL**y
  - \9§ Ln **}9jc L_ §}_@«§_nj** ennc e_cOy
  - \9§ **°OeT9}O** \n«eL °O O²qOH§ nj O9H\ _LOy

Our objectives

[illegible]

Our objectives

[illegible]

Our objectives

ª jLO} §9jL §\0 °OeT9}0 §}«H§«}0 _j §_ X0j0}9e hnL0eI

- _H\ 9X0j§ 9}0 hn}0 Hnhq0§_§_®0¤qnq«e9}y
- §\0 h9}c0§ °Oee Hnjj0H§0Ly
- \9§ Ln }9jc L_ §}_@«§_nj ennc e_c0y
- \9§ °OeT9}0 \n«eL °0 0²q0H§ nj 09H\ _L0y

q ? i / Q i ? Q b 2 2 p 2 M K 2 i 2 M Q B ; M 2 M 2 Q m b K ` F 2 i

Wn9eI ª jL0} H0}§9_j **Hnjj0H§_®_§´ HnjL_§_nj** nj §\0 h9}c0§J 9ee §9@e0 n«§Hnh0 @0\9®0 _j 9 **H0}§9_j °9´** §\9§ }0¼0H§ **»§j0 nT 9X0j§** u

â ª e§_h9§0e´ \0eq_jX h9}c0§ L0 _Xj0} §n @0§§0} 9j9e´¶0 9jL 0j\9jH0 h9}c0§ 0¹H_0jH´J 0uXuJ }0®09e_jX _j0¹H_0jH´J _L0j§_T´_jX L_ 9L®9j§9X0L X}n«q J 0§Hu

Contents

Literature

- ª j_Tn}he´ }9jLnh q}OTO}OjHO
  - Number of stable matchings ($\frac{n \log n}{e}$), optimal and pessimal average ranks of each side (Pittel 1989)
  - Number of stable partners; "law of hyperbola": product of average ranks of the two sides n (Pittel 1992)
  - Unbalanced markets: short side advantage (Ashlagi, Kanoria, and Leshno 2017; Cai and Thomas 2022)
- g9}cO§ °_§\ q«@e_H Hn}O r^h hn}e_H9 9jL g9\L_9j ©·oU¡ bna_h9 9jL p9§\9c ©··k¡ 8 \e9X_J ?}9®O}h9jJ 9jL [9 _L_h ©·oVJO§Hus
- K_ §}_@«§_nj nT h9§H\ H\9}9H§O}_ §_H r g Oj¶Oe ©·oU¡ pŞ c_ ©·o¢s
  - Probability of a pair being matched
  - General preference model, many agents of each type

Preparation: fitness of agents and contiguity of market

Fitness of agents

Question

* Mr2 +? ` +i2`Bx2 2t@ Mi2 +QKT2iBiBp2M2bb Uó
K `F2iM/2T2M/2M+2 Q7 HBx2/ bi #H2 K i+?BM;\

Fitness of agents

Question

* M r2 +? ` +i2`Bx2 2t@ Mi2 +QKT2iBiBp2M2bb Uó
K `F2BiM/2T2M/2M1 Q27 HBx2/ bi #H2 K i+?BM;\

Example (Public fitness)

$$\hat{A} = \begin{array}{c} \\ \text{Amy} \\ \text{Betty} \\ \text{Cindy} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Dan} & \text{Evan} & \text{Fran} \end{array} \\ \begin{pmatrix} 1/2 & 1/3 & 1/6 \\ 1/2 & 1/3 & 1/6 \\ 1/2 & 1/3 & 1/6 \end{pmatrix} \end{array} \qquad \hat{B} = \begin{array}{c} \\ \text{Dan} \\ \text{Evan} \\ \text{Fran} \end{array} \begin{array}{c} \begin{array}{ccc} \text{Amy} & \text{Betty} & \text{Cindy} \end{array} \\ \begin{pmatrix} 1/5 & 3/10 & 1/2 \\ 1/5 & 3/10 & 1/2 \\ 1/5 & 3/10 & 1/2 \end{pmatrix} \end{array}$$

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– \9§ 9@n«§ h9}c0§ °_§\ jnj]q«@e_H »§j0 y

Fitness through mutual scaling

Observation

_2b+ HBM; i?Â 2 `QMBb UQ7X 2 X+diag( )Â MB diag( )B̂ rBi?

; 2 $\mathbb{R}^n_+$ V ? b MQ BKT +i QM i?2 T`272`2M+2 KQ/2HX

Fitness through mutual scaling

Observation

_ 2 b + H B M ; i ?2 ` Q B b U Q 7 2 X diag( )Â M B diag( )B̂ r B i ?
; 2 $\mathbb{R}^n_+$ V ? b M Q B K T + i Q M i ? 2 T ` 2 7 2 ` 2 M + 2 K Q / 2 H X
h ? 2 ` 2 2 t B 2 b i b 2 M i B H H v 2 b + M B [m 2 ; b m + ? i ? i

$$M = nA \quad B^{>}$$

B b / Q m # H v b i Q + ? b i B + U ? p B M ; m M B i ` Q r M / + Q H m

Fitness through mutual scaling

Observation

_ 2 b + H B M ; i ? $\hat{A}$ ` Q $\hat{B}$ b U Q 7 X 2 X diag( ) $\hat{A}$ M B diag( ) $\hat{B}$ r B i ?
; 2 $\mathbb{R}^n_+$ V ? b M Q B K T + i Q M i ? 2 T ` 2 7 2 ` 2 M + 2 K Q / 2 H X
h ? 2 ` 2 2 t B b i b 2 M i B H H v 2 m + M B [m 2 ; b m + ? i ? i

$$M = nA \quad B^{>}$$

B b / Q m # H v b i Q + ? b i B + U ? p B M ; m M B i ` Q r M / + Q H m

- M I h «§«9e h 9§}_²
- 9jL I r9j§_]s»§j0 n⊺ §\0 °nh0j 9jL h0j

Example: public popularity (revisited)

$$\hat{A} = \begin{pmatrix} & \text{Dan} & \text{Evan} & \text{Fran} \\ \text{Amy} & 1/2 & 1/3 & 1/6 \\ \text{Betty} & 1/2 & 1/3 & 1/6 \\ \text{Cindy} & 1/2 & 1/3 & 1/6 \end{pmatrix} \qquad \hat{B} = \begin{pmatrix} & \text{Amy} & \text{Betty} & \text{Cindy} \\ \text{Dan} & 1/5 & 3/10 & 1/2 \\ \text{Evan} & 1/5 & 3/10 & 1/2 \\ \text{Fran} & 1/5 & 3/10 & 1/2 \end{pmatrix}$$

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$$\hat{A} = \begin{array}{c} \\ \text{Amy} \\ \text{Betty} \\ \text{Cindy} \end{array} \!\!\! \begin{array}{c} \begin{array}{ccc} \text{Dan} & \text{Evan} & \text{Fran} \end{array} \\ \begin{pmatrix} 1/2 & 1/3 & 1/6 \\ 1/2 & 1/3 & 1/6 \\ 1/2 & 1/3 & 1/6 \end{pmatrix} \end{array} \qquad \hat{B} = \begin{array}{c} \\ \text{Dan} \\ \text{Evan} \\ \text{Fran} \end{array} \!\!\! \begin{array}{c} \begin{array}{ccc} \text{Amy} & \text{Betty} & \text{Cindy} \end{array} \\ \begin{pmatrix} 1/5 & 3/10 & 1/2 \\ 1/5 & 3/10 & 1/2 \\ 1/5 & 3/10 & 1/2 \end{pmatrix} \end{array}$$

$$\propto \begin{array}{c} \text{Amy} \\ \text{Betty} \\ \text{Cindy} \end{array} \begin{pmatrix} 1/2 \\ 1/3 \\ 1/5 \end{pmatrix} \qquad \propto \begin{array}{c} \text{Dan} \\ \text{Evan} \\ \text{Fran} \end{array} \begin{pmatrix} 2 \\ 3 \\ 6 \end{pmatrix}$$

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Connectivity of the market

Example (Sub-markets)

Gnj _L0} $A = B = \frac{2}{n}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ u ¦\0 h9}c0§ L0Hnhqn 0 _j§n §°n _jL0q0jL0j§ «@]h9}c0§ u

Connectivity of the market

Example (Sub-markets)

[illegible] $A = B = \frac{2}{n}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ [illegible]

[illegible]

Connectivity of the market

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Gnj _L0} $A = B = \frac{2}{n}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ u ¦\0 h9}c0§ L0Hnhqn 0 _j§n §°n _jL0q0jL0j§ «@]h9}c0§ u

¯0 H9j nje´ \nq0 §n H\9}9H§0}_¶0 h9}c0§ §\9§ 9}0 ō°0ee Hnjj0H§0LŸu

Assumption (Connectivity)

h?2`2 2 tCB<b1 b B M/2 T 2 M h2bMni +Q?7 i? i

$$\frac{a_{ij}}{a_{ij'}}, \frac{b_{ji}}{b_{ji'}} \quad C \qquad \forall i, i', j, j'.$$

- ¦\0 «j_Tn}h H9 0 _ §\0 q0H_9e H9 0 °\0j $C = 1$

Connectivity of the market

Example (Sub-markets)

[illegible] $A = B = \frac{2}{n}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ [illegible]

[illegible]

Assumption (Connectivity)

[illegible]

$$\frac{a_{ij}}{a_{ij'}}, \frac{b_{ji}}{b_{ji'}} \le C \qquad \forall i, i', j, j'.$$

- [illegible] $C = 1$
- [illegible] A, B [illegible] M [illegible]

Results: characterizing welfare distribution

Welfare characterization

[illegible]

- [illegible]
- [illegible]

Welfare characterization

¯0 h09 «}0 9§_ T9H§_nj nT 9j 9X0j§ @´ \0} **}9jc]§n]r9j§_]s»§j0 r| ¦Ss }9§_nI**

- W_®0j 9j 9X0j§l »§j0 J h9ee0} }9jc h09j \9qq_0}
- W_®0j 9j 9X0j§l }9jcJ e9}X0} r9j§_]s»§j0 h09j \9qq_0}

Theorem (Informal)

q?T- BM 2p2`v bi #H2 K i+?BM;- i?2 7QHHQrBM;b ?Q

1 S`Q/m+i Q7 irQ bB/2bö p2` ;n2c_h6b Bb +HQb2 iQ

â h` /2@Qz #2ir22M i?2 irQ bB/2b

Welfare characterization

¯0 h09 «}0 9§_ T9H§_nj nT 9j 9X0j§ @´ \0} **}9jc]§n]r9j§_]s»§j0 r| ¦Ss }9§_nI**

- W_®0j 9j 9X0j§l »§j0 J h9ee0} }9jc h09j \9qq_0}
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q?T- BM 2p2`v bi #H2 K i+?BM;- i?2 7QHHQrBM;b ?

1 S`Q/m+i Q7 irQ bB/2bö p2` ;n2c_h6b Bb +HQb2 iQ

â h` /2@Qz #2ir22M i?2 irQ bB/2b

2 1KTB`B+ H /Bbi`B#miBQM Q7 _h6b QM 2 +? bB/2

â MiB@+QM+2Mi` iBQM /m2 iQ bi #BHBiv +QMbi

Example: the uniform case

Corollary (Informal, uniform case)

Example: the uniform case

N9H\ 9X0j§l q}0T0}0jH0 _ 9 **«j_Tn}he´ }9jLnh** n}L0}_jX nT §\0 nqqn _§0 _L0u

Corollary (Informal, uniform case)

q?T- BM 2p2`v bi #H2 K i+?BM;- i?2 7QHHQrBM;b ?

1. S`Q/m+i Q7 irQ bB/`2 bMiFbb 2+`H;@2b2iQ Q7 ?vT2`# QRHNN S

2. 1KTB`B+ H /Bbi`B#miBQM Q7 ?Qb7B/2 Bb +HQb2 iQ 2t

Example: the uniform case

N9H\ 9X0j§l q}0T0}0jH0 _ 9 **«j_Tn}he´ }9jLnh** n}L0}_jX nT §\0 nqqn _§0 _L0u

Corollary (Informal, uniform case)

q ? T - B M 2 p 2 ` v b i # H 2 K i + ? B M ; - i ? 2 7 Q H H Q r B M ; b ?

1. S ` Q / m + i Q 7 i r Q b B / ` 2 b M F B b 2 + ` H ; @ 2 b 2 i Q 7 ? v T 2 ` # Q H

2. 1 K T B ` B + H / B b i ` B # m i B Q M Q 7 B / 2 B b + H Q b 2 i Q 2 t

Example: the uniform case

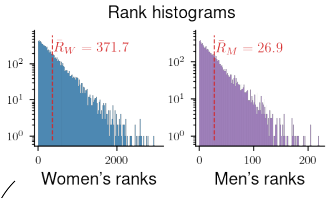
N9H\ 9X0j§ļ q}OTO}0jHO _ 9 **«j_Tn}he´ }9jLnh** n}L0}_jX nT §\0 nqqn _§0 _L0u

Corollary (Informal, uniform case)

q?T- BM 2p2`v bi #H2 K i+?BM;- i?2 7QHHQrBM;b ?Q

1 S`Q/m+i Q7 irQ bB/`2bMdFBbp 2+`H;Q2bu2HiQrQ7 ?vT2`# QRHNMS

2 1KTB`B+ H /Bbi`B#miBQM Q7 ?Q7B/2 Bb +HQb2 iQ 2t



1t KTH2, i?2 mMB7Q`K + b2

1 +? ;2Miöb T`27m2`2M+2 Kb v Q`/2` BM ; Q7 i?2 QTTQ

[redacted]

q?T- BM 2p2`v bi #H2 K i+?BM;- i?2 7QHHQrBM;b ?Q

- S`Q/m+i Q7 irQ bB/2 MmK#2` Q7 #HQ+FBM; T B`b 2t +2 2/b i?2 M2` i?Q`BGHi r Q7 ?vT2`# QHB+ N?S
- 1KTB`B+ H /Bbi`B#miBQM Q7 i?2 M+HmbBQM Bb +HQb2 iQ 2t

1t KTH2, i?2 mMB7Q`K+ b2

1 +? ;2Miöb T`27 2`2M+2b Q`/2`BM; Q7 i?2 QTTQ

[redacted]

q?T- BM 2p2`v bi #H2 K i+?BM;- i?2 7QHHQrBM;b ?Q

- S`Q/m+i Q7 irQ bB/2 M+B2b 2`H;2b iQ 2 i?2 Q7 ?vT2`#QH
- 1KTB`B+ H /Bbi`B#miBQM Q7 ?Q b7B/2 Bb +HQb2 iQ 2t

1t KTH2, i?2 mMB7Q`K + b2

1 +? ;2Miöb T`27m2M+B2b hvQ`M2/ QK M; Q7 i?2 QTTQ

[redacted]

q?T- BM 2p2`v bi #H2 K i+?BM;- i?2 7QHHQrBM;b ?Q

R S`Q/m+i Q7 irQ bB/2 bm`Tb 2`H;2b iQ UHi r Q7 ?vT2`# QHN N!S

k 1KTB`B+ H /Bbi`B#miBQM Q7 ?Q/B2b Bb +HQb2 iQ 2t

1t KTH2, i?2 mMB7Q`K+ b2

1 +? ;2Miöb T`27n`2M+2 K Q`2/ÈK; Q7 i?2 QTTQ

[redacted]

q?T- BM 2p2`v bi #H2 K i+?BM;- i?2 7QHHQrBM;b ?Q

R S`Q/m+i Q7 irQ bB/`2 bMöFBbp 2+`H;Q2b2 UHi r Q7 ?vT2`# QRHNVS

k 1KTB`B+ H /Bbi`BM#FQbMB2QM+?Qb7B/2 Bb +HQb2 iQ 2t

1t KTH2, i?2 mMB7Q`K + b2

1 +? ;2Miöb T`27m2`M2B7+Q2`Kb hvQ`M/2Q`BKM; Q7 i?2 QTTQ

[redacted]

q?T- BM 2p2`v bi #H2 K i+?BM;- i?2 7QHHQrBM;b ?Q

- S`Q/m+i Q7 irQ bB/`2 MöFBbp 2+`H;Q2b2 UHi r Q7 ?vT2`# QHN NS
- 1KTB`B+ H /Bbi`` BM#FQbM2 Q M+ ?Qb7B/2 Bb +HQb2 iQ 2t

1t KTH2, i?2 mMB7Q`K + b2

1 +? ;2Miöb T`2 7m`2 M2B7+Q2`K b Hv Q`/2` BK M; Q7 i?2 QTTQ

[redacted]

q?T- BM 2p2`v bi #H2 K i+?BM;- i?2 7QHHQrBM;b ?Q

R S`Q/m+i Q7 irQ bB/`2 M ö B b 2+`H;Q2b2 i Q UHi r Q7 ?vT2`# QRHNVS

k 1KTB`B+ H /Bbi`B#miBQM Q7 M2 M ?Q7B/2 Bb +HQb2 iQ 2t

[redacted]

LQM@2tTQM2MiB H `

6 BHm`2 Q7 óH r Q7 ?v

Discussion: stable matchings vs RSD

(a) Woman-optimal stable matching

(b) RSD

- [illegible]

Discussion: stable matchings vs RSD

(a) Woman-optimal stable matching

(b) RSD

- ¦ \0 9L®9j§9X0L _L0 \9 §\0 **9h0 9®0}9X0 }9jc** _j @n§\ h9§H_jX rHn«qnj HneeOH§n}sJ @«§ ®0}´ **L_¸0}0j§ L_ §}_@«§_nj**
- ¦ \0 L0T0}}0L 9HH0q§9jH0 h0H\9j_ h _ ´**h h0§}_H**I n}L0}_jX nT q}nqn _jX Ln0 jn§ h9§§0}¡ |~K _ jn§I n}L0}_jX h9§§0}

Discussion: stable matchings vs RSD

(a) Woman-optimal stable matching

(b) RSD

- ¦ \O 9L®9j§9XOL _LO \9 §\O **9h0 9®O}9XO }9jc** _j @n§\ h9§H_jX rHn«qnj HneeOH§n}sJ @«§ ®O}´ **L_¸O}Oj§ L_ §}_@«§_nj**
- ¦ \O LOTO}}OL 9HHOq§9jHO h OH\9j_ h _ ´**h h0§}_H**I n}LO}_jX nT q}nqn _jX Ln0 jn§ h9§§O}¡ |~K _ jn§I n}LO}_jX h9§§O}

Discussion: stable matchings vs RSD

(a) Woman-optimal stable matching

(b) RSD

- ¦ \0 9L®9j§9X0L _L0 \9 §\0 **9h0 9®0}9X0 }9jc** _j @n§\ h9§H_jX rHn«qnj Hnee0H§n}sJ @«§ ®0}´ **L_¸0}0j§ L_ §}_@«§_nj**
- ¦ \0 L0T0}}0L 9HH0q§9jH0 h0H\9j_ h _ ´**h h0§}_HI** n}L0}_jX nT q}nqn _jX Ln0 jn§ h9§§0}¡ |~K _ jn§I n}L0}_jX h9§§0}

  â g9² 0j§}nq´ \0«}_ §_H I _j §9@e0 h9§H_jX J **9®0}9X0 }9jc H9q§«}0 ō9ee _jTnŸ**¡ _j |~KJ §\0}0 _ 0²§}9 _jTn

Discussion: stable matchings vs RSD

(a) Woman-optimal stable matching

(b) RSD

- [illegible]
- [illegible]

  [illegible]
- [illegible]

Sketch of analysis

An equivalent preference model

|9jc _ L_ H}0§0 Ś \9}L §n °n}c °_§\J « 0 9 Hnj§_j«n« q}n²´I

- N9H\ °nh9j i X0j0}9§0 9 ®9e«0 X_{ij} Exp(na_{ij}) _jL0q0jL0j§e´ Tn} 09H\ h9j j

An equivalent preference model

|9jc _ L_ H}0§0 Ś \9}L §n °n}c °_§\J « 0 9 Hnj§_j«n« q}n²´I

- N9H\ °nh9j i X0j0}9§0 9 ®9e«0 X_{ij} Exp(na_{ij}) _jL0q0jL0j§e´ Tn} 09H\ h9j j
- ¯nh9j i q}0T0} h9j j §n j' () $X_{ij} < X_{ij'}$

An equivalent preference model

|9jc _ L_ H}0§0 Ś \9}L §n °n}c °_§\J « 0 9 Hnj§_j«n« q}n²´I

- N9H\ °nh9j i X0j0}9§0 9 ®9e«0 X_{ij} Exp(na_{ij}) _jL0q0jL0j§e´ Tn} 09H\ h9j j
- ¯nh9j i q}0T0} h9j j §n j^0 () $X_{ij} < X_{ij^0}$
- g0jl q}0T0}0jH0 9}0 X0j0}9§0L 9j9enXn« e´

M 2[mBp H2Mi T`272`2M+2 KQ/2H

_ MF Bb /Bb+`2i2 ? `/ iQ rQ`F rBi?- mb2 +QMiBMm

1 +? rQK ;M2M2` i2bH X_{ij} 2 $\mathrm{Exp}(na_{ij})$ BM/2T2M/2MiHv
K Nj

qQK iMT`272`bj kQM(l) $X_{ij} < X_{ij^0}$

J2Möb T`272`2M+2b `2 ;2M2` i2/ M HQ;QmbHv

M 2[mBp H2Mi T`272`2M+2 KQ/2H

- MF Bb /Bb+`2i2 ? `/ iQ rQ`F rBi?- mb2 +QMiBMm
  - 1 +? rQK ;M2M2` i2b Hm2 $X_{ij} = \mathrm{Exp}(na_{ij})$ BM/2T2M/2MiHv K Nj
  - qQK iMT`272`b KQ/2H j iQ j' B7 $X_{ij} < X_{ij'}$
  - J2Möb T`272`2M+2b `2 ;2M2` i2/ M HQ;QmbHv
- aK HH2` p Hm2 M/ bK H!H2`b KU HMiB2`@`V MiFX2 abm{+2b 2KTB`B+ H /Bbi`B#miBQM Q7 p Hm2b

1biBK i2 HBF2HB?QQ/ Q7 bi #BHBiv

*QMbf B/2$ Amy $ Dan, Betty $ Evan, Cindy $ Frang

1biBK i2 HBF2HB?QQ/ Q7 bi #BHBiv

*QMbB/2` Amy $ Dan, Betty $ Evan, Cindy $ Frang

G2i $x_1, \ldots, x_3$ M/ $y_1, \ldots, y_3$ #2 i?2 `2 HBx2/ p Hm2b

1biBK i2 HBF2HB?QQ/ Q7 bi #BHBiv

*QMbB/2\$ Amy $ Dan; Betty $ Evan; Cindy $ Frang

G 2 $x_1; \ldots; x_3$ My $y_1; \ldots; y_3$ #2 i?2 `2 HBx2/ p Hm2b

GBF2HB?QQ/ Q7 bi #BHBiv ;Bp2M p Hm2b

$$(1 - P(\text{Amy}; \text{Evan} \ \#\text{HQ}))(1 - P(\text{Amy}; \text{Fran} \ \#\text{HQ})) + F(1 - P(\text{Cindy}; \text{Evan} \ \#\text{HQ})) + F$$

$$= (1 - F_{na_{12}}(x_1)F_{nb_{21}}(y_2))(1 - F_{na_{13}}(x_1)F_{nb_{31}}(y_3)) \quad (1 - F_{na_{32}}(x_3)F_{nb_{23}}(y_2))$$

$$\prod_{i \neq j}(1 - n^2 a_{ij} b_{ji} x_i y_j) \quad \prod_{i;j}(1 - nm_{ij} x_i y_j) \quad \exp(-nx^{>} My)$$

r?2 `$F(z) = 1 - e^{-z}$ /2MQi2b 2tTQM2MiB H *.6

1biBK i2 HBF2HB?QQ/ Q7 bi #BHBiv

* QMb fBMi2 Amy $ Dan, Betty $ Evan, Cindy $ Fran.
G 2Mi $x_1; \ldots ; x_3$ M/ $y_1; \ldots ; y_3$ #2 i?2 `2 HBx2/ p Hm2b
G BF2HB?QQ/ Q7 bi #BHBiv ;Bp2M p Hm2b

$$(1 - P(\text{Amy}; \text{Evan} \; \#\,H\,Q))(1 - P(\text{Amy}; \text{Fran} \; \#\,H\,Q)) + F(1 - P(\text{Cindy}; \text{Evan} \; \#\,H\,Q)) + F$$

$$= (1 - F_{na_{12}}(x_1)F_{nb_{21}}(y_2))(1 - F_{na_{13}}(x_1)F_{nb_{31}}(y_3)) \quad (1 - F_{na_{32}}(x_3)F_{nb_{23}}(y_2))$$

$$\prod_{i \neq j} (1 - n^2 a_{ij} b_{ji} x_i y_j) \quad \prod_{i;j} (1 - nm_{ij} x_i y_j) \quad \exp(-nx^{>} My) \quad \exp(-\sum x_i y_j)$$

r?2`2 $F(z) = 1 - e^{-z}$ /2MQi2b 2tTQM2MiB H *.6
lb2 +QMiB;mBiv bbmKTiBQM $nx^{>} My = \sum_{i;j} x_i y_j$?Q7rQ` HBF2Hv K
â > TT2Mb $x;y$?2M2 #Qi? BM i?2 T`BM+BTH 2B;2M

1 b i B K i 2 H B F 2 H B ? Q Q / Q 7 b i # B H B i v

* Q M b B / 2 ` f Amy $ Dan, Betty $ Evan, Cindy $ Frang

G 2 $x_1, \ldots, x_3$ M $y_1, \ldots, y_3$ # 2 i ? 2 ` 2 H B x 2 / p H m 2 b

G B F 2 H B ? Q Q / Q 7 b i # B H B i v ; B p 2 M p H m 2 b

$$(1 - P(\text{Amy}; \text{Evan} \ \# H \ \mathbb{Q})) + F(1 - P(\text{Amy}; \text{Fran} \ \# H \ \mathbb{Q})) + F(1 - P(\text{Cindy}; \text{Evan} \ \# H \ \mathbb{Q})) + F$$

$$= (1 - F_{na_{12}}(x_1)F_{nb_{21}}(y_2))(1 - F_{na_{13}}(x_1)F_{nb_{31}}(y_3)) \cdots (1 - F_{na_{32}}(x_3)F_{nb_{23}}(y_2))$$

$$\prod_{i \neq j} (1 - n^2 a_{ij} b_{ji} x_i y_j) \quad \prod_{i;j} (1 - nm_{ij} x_i y_j) \quad \exp(-nx^{\top} My) \quad \exp(-\sum x_i y_j)$$

r ? 2 ` F 2 $(z) = 1 - e^{-z}$ / 2 M Q i 2 b 2 t T Q M 2 M i B H * . 6

l b 2 + Q M i B ; m B i v b b m K $x^{\top} M y$ i B Q M $\sum_{i,j} x_i y_j$? Q 7 r Q ` H B F 2 H v K

â > T T 2 M b $x; y$ 2 M 2 # Q i ? B M i ? 2 T ` B M + B T H 2 B ; 2 M b

A M T Q b i 2 ` B Q ` / B i B Q M H m T Q M b i # B H B i v $\exp(\sum y_j)$ H Q Q F b K T H 2 b

1 b i B K i 2 H B F 2 H B ? Q Q / Q 7 b i # B H B i v

* Q M b B / 2 $ Amy; Dan $ Betty; Evan $ Cindy; Fran

G 2 i $x_1; :::; x_3$ M / $y_1; :::; y_3$ # 2 i ? 2 ` 2 H B x 2 / p H m 2 b

G B F 2 H B ? Q Q / Q 7 b i # B H B i v ; B p 2 M p H m 2 b

$$(1 - P(\text{Amy}; \text{Evan} \ \# H \ Q)) F (1 - P(\text{Amy}; \text{Fran} \ \# H \ Q)) + F (1 - P(\text{Cindy}; \text{Evan} \ \# H \ Q)) + F$$

$$= (1 - F_{na_{12}}(x_1)F_{nb_{21}}(y_2))(1 - F_{na_{13}}(x_1)F_{nb_{31}}(y_3)) \quad (1 - F_{na_{32}}(x_3)F_{nb_{23}}(y_2))$$

$$\prod_{i \neq j} (1 - n^2 a_{ij} b_{ji} x_i y_j) \quad \prod_{i;j} (1 - nm_{ij} x_i y_j) \quad \exp(-nx^{\top} My) \quad \exp(-\sum x_i y_j)$$

r ? 2 ` 2 $F(z) = 1 - e^{-z}$ / 2 M Q i 2 b 2 t T Q M 2 M i B H * . 6

l b 2 + Q M i B ; m B i v b b m K T i B Q M $x^{\top} My$ i Q $\sum_{i;j} x_i y_j$ Q ` H B F 2 H v K

â > T T 2 M b x ; y 2 M ` 2 # Q i ? B M i ? 2 T ` B M + B T H 2 B ; 2 M b

A M T Q b i 2 ` B Q ` / B i B Q M H m T Q M b i # B H B i v $\exp(-\sum x_i y_j)$ H Q Q F b
b K T H 2 b

l b 2 b i M / ` / + Q M + 2 M i ` i B Q M B M 2 [m H B i B 2 b M / m

.Bb+mbbBQM

amKK `v, *? ` +i2`Bx iBQM Q7 ` MFfr2H7 `2 /Bbi

- :HQ# H i` /2@Qz #2ir22M i?2 bB/2b
- AMi`BMbB+ [m HBiv Q7 ;2Mib
- 1tTQM2MiB H ?BbiQ;` K Q7 _h6

Discussion

- ~« h h 9}´I G\9}9H§0}_¶9§_nj nT }9j c¤°0eT9}0 L_ §}_@«§_nj _j §9@e0 h 9§H_j X
  - Global trade-off between the sides
  - Intrinsic quality of agents
  - Exponential histogram of RTF
- |0 «e§ 9jL 9j9e´ _ 0²§0jL §n 9e h n § §9@e0 h 9§H_j X J _jHe«L_j X 9e h n § @9e9 j H0L h 9}c0§ r°_§\ «@e_j09} _h @9e9 j H0s

Discussion

- ~« h h 9}´I G\9}9H§0}_¶9§_nj nT }9j c¤°0eT9}0 L_ §}_@«§_nj _j §9@e0 h 9§H_j X
  - Global trade-off between the sides
  - Intrinsic quality of agents
  - Exponential histogram of RTF
- |0 «e§ 9j L 9j 9e´ _ 0²§0j L §n 9e h n § §9@e0 h 9§H_j X J _j He«L_j X 9e h n § @9e9j H0L h 9}c0§ r°_§\ «@e_j 09} _h @9e9j H0s
  â dnnc 9§ e9}X0 «@]h 9}c0§
- ¦\0 Hnj j0H§_®_§´ Hnj L_§_nj H9j @0 }0e9²0L

Open directions

- g 9j ´]§n]nj0 9jL h9j ´]§n]h9j ´ h9§H_jX J h«e§_] _L0L h9§H_jX J Hn}}0e9§0L q}0T0}0jH0 J 0§Hu
- W0j0}9e_¶9§_nj nT §\0 Hnjj0H§_®_§´ HnjL_§_nj r9j9enXn« §n 0²q9j _nj nT X}9q\ s
- N h q_}_H9e 0®_L0jH0 r0uXuJ T}n h i | g ps
- N ¹ H_0j§ 9eXn}_§\ h Tn} _jT0}}_jX¤e09}j_jX »§j0 9jL Hnjj0H§_®_§´ T}n h 2 t @ Tn@ b}®9§_nj rq}0T0}0jH0 9jL n«§Hn h0 s

Open directions

- g 9j ´]§n]nj0 9jL h9j ´]§n]h9j ´ h9§H_jX J h«e§_] _L0L h9§H_jX J Hn}}0e9§0L q}0T0}0jH0 J 0§Hu
- W0j0}9e_¶9§_nj nT §\0 Hnjj0H§_®_§´ HnjL_§_nj r9j9enXn« §n 0²q9j _nj nT X}9q\ s
- N h q_}_H9e 0®_L0jH0 r0uXuJ T}n h i | g ps
- N ¹ H_0j§ 9eXn}_§\ h Tn} _jT0}}_jX¤e09}j_jX »§j0 9jL Hnjj0H§_®_§´ T}n h 2 t @ Tn@ b}0 9§_nj rq}0T0}0jH0 9jL n«§Hn h 0 s

Wn9eI [0eq _L0j§_T´ h9}c0§ T9_e«}0 9jL 0j\9jH0 h9}c0§ 0¹H_0jH´

Open directions

- [illegible]
- [illegible]
- [illegible]
- [illegible]

[illegible]

Thank you!