College Admissions with Housing Quotas

Denis Sokolov
FairPlay team

CIRM

December 14th, 2023

Roadmap

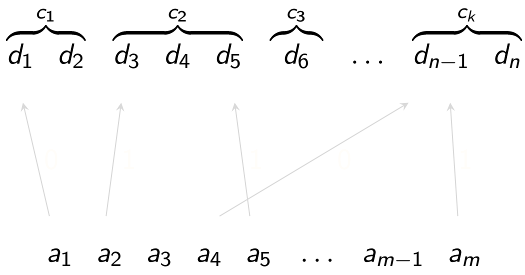
- General framework: possible applications
- College housing crisis
- Stable matching: six types of blocking contracts
- *Take-House-from-Applicant*-Stability
- *Not-Compromised-Request-from-One-Agent*-Stability
- Blocking domination: IP solution
- Concluding remarks

Roadmap

- General framework: possible applications
- College housing crisis
- Stable matching: six types of blocking contracts
- *Take-House-from-Applicant*-Stability
- *Not-Compromised-Request-from-One-Agent*-Stability
- Blocking domination: IP solution
- Concluding remarks

General Framework

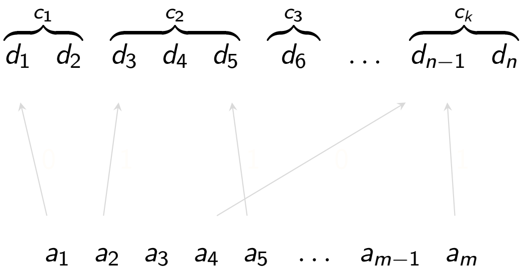
This paper considers two-sided matching markets of the following structure:



- c: **quota** q_c;
- d: **quota** q_d, and **strict preferences** over $\cup\{a\} \cup \varnothing$;
- a: **strict preferences** over $(\cup\{d\} \times \{0, 1\}) \cup \varnothing$.

General Framework

This paper considers two-sided matching markets of the following structure:



- c: **quota** q_c;
- d: **quota** q_d, and **strict preferences** over $\cup\{a\} \cup \varnothing$;
- a: **strict preferences** over $(\cup\{d\} \times \{0, 1\}) \cup \varnothing$.

General Framework

This paper considers two-sided matching markets of the following structure:

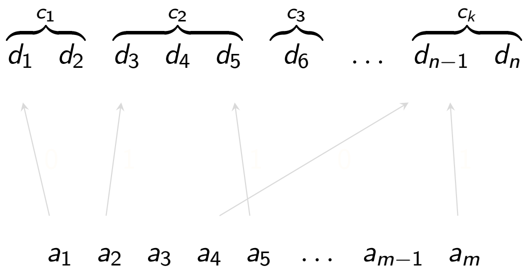
$$\overbrace{d_1 \quad d_2}^{c_1} \quad \overbrace{d_3 \quad d_4 \quad d_5}^{c_2} \quad \overbrace{d_6}^{c_3} \quad \ldots \quad \overbrace{d_{n-1} \quad d_n}^{c_k}$$

$$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad \ldots \quad a_{m-1} \quad a_m$$

- c: **quota** q_c;
- d: **quota** q_d, and **strict preferences** over $\cup\{a\} \cup \varnothing$;
- a: **strict preferences** over $(\cup\{d\} \times \{0, 1\}) \cup \varnothing$.

General Framework

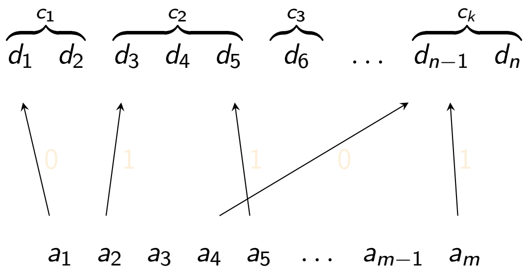
This paper considers two-sided matching markets of the following structure:



- c: **quota** q_c;
- d: **quota** q_d, and **strict preferences** over $\cup\{a\} \cup \varnothing$;
- a: **strict preferences** over $(\cup\{d\} \times \{0, 1\}) \cup \varnothing$.

General Framework

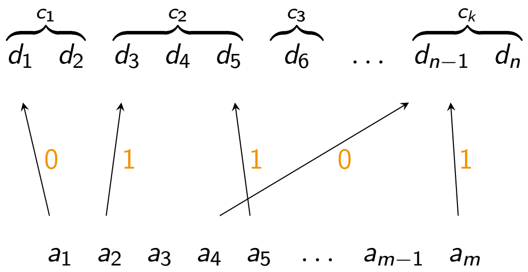
This paper considers two-sided matching markets of the following structure:



- c: **quota** q_c;
- d: **quota** q_d, and **strict preferences** over $\cup\{a\} \cup \varnothing$;
- a: **strict preferences** over $(\cup\{d\} \times \{0,1\}) \cup \varnothing$.

General Framework

This paper considers two-sided matching markets of the following structure:



- c: **quota** q_c;
- d: **quota** q_d, and **strict preferences** over $\cup\{a\} \cup \varnothing$;
- a: **strict preferences** over $(\cup\{d\} \times \{0, 1\}) \cup \varnothing$.

Japan: Hospital/Residents Matching with Regions

Under the Japanese medical residency matching market (Kamada and Kojima) we have:

- c is a **region** that can accept no more than q_c doctors;
- d is a **hospital** that can accept no more than q_d doctors;
- a is a **doctor** with strict preferences over $\cup\{d\} \cup \varnothing$.

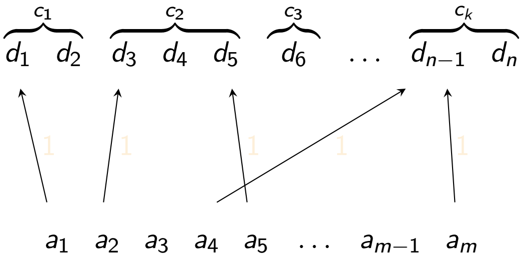
$$\overbrace{d_1 \quad d_2}^{c_1} \quad \overbrace{d_3 \quad d_4 \quad d_5}^{c_2} \quad \overbrace{d_6}^{c_3} \quad \ldots \quad \overbrace{d_{n-1} \quad d_n}^{c_k}$$

$$a_1 \quad a_2 \quad a_3 \quad a_4 \quad a_5 \quad \ldots \quad a_{m-1} \quad a_m$$

Japan: Hospital/Residents Matching with Regions

Under the Japanese medical residency matching market (Kamada and Kojima) we have:

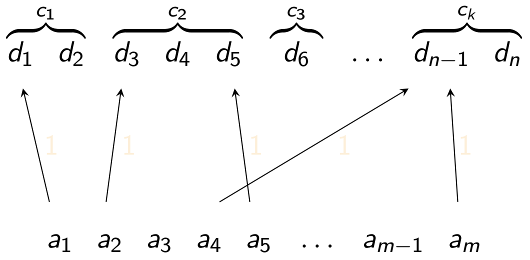
- c is a **region** that can accept no more than q_c doctors;
- d is a **hospital** that can accept no more than q_d doctors;
- a is a **doctor** with strict preferences over $\cup\{d\} \cup \varnothing$.



Japan: Hospital/Residents Matching with Regions

Under the Japanese medical residency matching market (Kamada and Kojima) we have:

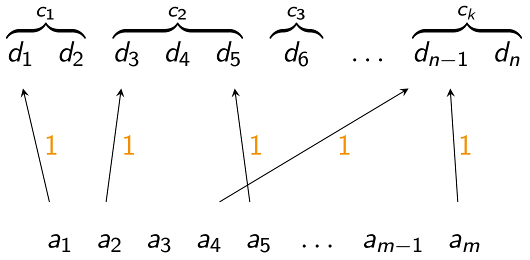
- c is a **region** that can accept no more than q_c doctors;
- d is a **hospital** that can accept no more than q_d doctors;
- a is a **doctor** with strict preferences over $\cup\{d\} \cup \varnothing$.



Japan: Hospital/Residents Matching with Regions

Under the Japanese medical residency matching market (Kamada and Kojima) we have:

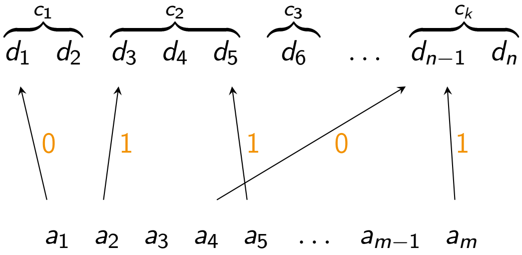
- c is a **region** that can accept no more than q_c doctors;
- d is a **hospital** that can accept no more than q_d doctors;
- a is a **doctor** with strict preferences over $\cup\{d\} \cup \varnothing$.



Hungary: College Admissions with State-Funding

Suppose that the government sets a national *quota for state-financed places* in each subject, then we have:

- c is a **subject** that has q_c *state-finances places* to distribute;
- d is a **department** that can accept no more than q_d applicants[1];
- a is an **applicant**.



[1] In Hungary some departments may impose lower quotas. Moreover, department preferences may have ties.

College Admissions with Housing

Suppose that each college has its own dormitory with a fixed amount of *beds* to give during the admissions process, then we have:

- c is a **college** that has q_c *beds* to distribute;
- d is a **department** that can accept no more than q_d applicants;
- a is an **applicant**.

College Admissions with Housing Quotas

A **college admissions market with housing quotas** (CAH) is a tuple $\Delta = \langle A, D, C, (P_a)_{a \in A}, (P_d, q_d)_{d \in D}, (q_c)_{c \in C} \rangle$.

Roadmap

- General framework: possible applications
- **College housing crisis**
- Stable matching: six types of blocking contracts
- *Take-House-from-Applicant*-Stability
- *Not-Compromised-Request-from-One-Agent*-Stability
- Blocking domination: IP solution
- Concluding remarks

College Housing Crisis: The U.S.

Bloomberg
Europe Edition

Sign In | Subscribe

● Live Now | Markets | Industries | Technology | Politics | Wealth | Pursuits | Opinion | Businessweek | Equality | Green | CityLab | Crypto | More

CityLab
Housing

Student Housing Crisis Offers Hard Lessons for U.S. Colleges

While undergrads at some U.S. campuses sleep in their cars, struggling schools have a surplus of dorm rooms.

The Washington Post
Democracy Dies in Darkness

HIGHER EDUCATION

Rising rents add to college students' scramble for affordable housing

A tight housing market and renewed desire for campus life have left some schools with long waiting lists for dorms

abc NEWS

VIDEO | LIVE | SHOWS | GUNS IN AMERICA | JAN. 6

Housing shortage, soaring rents squeeze US college students

College students squeezed by a massive housing shortage and rising rents are living in motels, commuting long distances or sleeping in their cars

College Housing Crisis: Europe

FRENCH EDUCATION • M campus Share

France's chronic student housing shortage blocks access to education for less well-off

France has only 380,000 spots in public or private residences for 2.7 million students. Many of them have to work in order to find a place to live.

telex

The pressing need for more student housing in Budapest

ENGLISH January 16. 2023. – 06:19 PM updated

Student housing

'Devastated' UK students forced to live in neighbouring cities in university accommodation crisis

A surge in the number of 18-year-olds combined with a lack of housing and landlords switching to Airbnb create a perfect storm

College Housing Crisis in Paris

*. . . considering the **significant shortage of student housing in the Paris region**. This lack of accommodation is particularly acute in the Créteil region, where no fewer than four universities and 130 schools are located. This area, which covers the **Seine-Saint-Denis region and all of eastern Paris, has only 5,300 CROUS housing places for 160,000 students**.*

Le Monde, June 10th, 2022

Motivating Example

There are 3 applicants, and 3 departments in 2 colleges: $c_1 = \{d_1, d_2\}$ and $c_2 = \{d_3\}$. Each department has a **unit capacity**. Preferences are:

a_1	a_2	a_3	$\{d_1$	$d_2\}$	$\{d_3\}$
d_1	d_1	d_1	a_1	a_1	a_1
d_2	d_2	d_2	a_2	a_2	a_2
d_3	d_3	d_3	a_3	a_3	a_3

The **unique stable matching** is $\{(a_1, d_1), (a_2, d_2), (a_3, d_3)\}$.

After admissions it turns out that **each college has exactly one bed**, both a_1 and a_2 always need a bed, while a_3 never needs one.

Applicant a_2 **does not get a bed** and drops out of the college.

Final matching: $\{(a_1, d_1, 1), (a_3, d_3, 0)\}$.

Motivating Example

There are 3 applicants, and 3 departments in 2 colleges: $c_1 = \{d_1, d_2\}$ and $c_2 = \{d_3\}$. Each department has a **unit capacity**. Preferences are:

a_1	a_2	a_3	$\{d_1$	$d_2\}$	$\{d_3\}$
d_1	d_1	d_1	a_1	a_1	a_1
d_2	d_2	d_2	a_2	a_2	a_2
d_3	d_3	d_3	a_3	a_3	a_3

The **unique stable matching** is $\{(a_1, d_1), (a_2, d_2), (a_3, d_3)\}$.

After admissions it turns out that **each college has exactly one bed**, both a_1 and a_2 always need a bed, while a_3 never needs one.

Applicant a_2 **does not get a bed** and drops out of the college.

Final matching: $\{(a_1, d_1, 1), (a_3, d_3, 0)\}$.

Motivating Example

There are 3 applicants, and 3 departments in 2 colleges: $c_1 = \{d_1, d_2\}$ and $c_2 = \{d_3\}$. Each department has a **unit capacity**. Preferences are:

a_1	a_2	a_3	$\{d_1$	$d_2\}$	$\{d_3\}$
$(d_1, 1)$	$(d_1, 1)$	$(d_1, 0)$	a_1	a_1	a_1
$(d_2, 1)$	$(d_2, 1)$	$(d_2, 0)$	a_2	a_2	a_2
$(d_3, 1)$	$(d_3, 1)$	$(d_3, 0)$	a_3	a_3	a_3

The **unique stable matching** is $\{(a_1, d_1), (a_2, d_2), (a_3, d_3)\}$.

After admissions it turns out that **each college has exactly one bed**, both a_1 and a_2 always need a bed, while a_3 never needs one.

Applicant a_2 **does not get a bed** and drops out of the college.

Final matching: $\{(a_1, d_1, 1), (a_3, d_3, 0)\}$.

Motivating Example

There are 3 applicants, and 3 departments in 2 colleges: $c_1 = \{d_1, d_2\}$ and $c_2 = \{d_3\}$. Each department has a **unit capacity**. Preferences are:

a_1	a_2	a_3	$\{d_1$	$d_2\}$	$\{d_3\}$
$(d_1, 1)$	$(d_1, 1)$	$(d_1, 0)$	a_1	a_1	a_1
$(d_2, 1)$	$(d_2, 1)$	$(d_2, 0)$	a_2	a_2	a_2
$(d_3, 1)$	$(d_3, 1)$	$(d_3, 0)$	a_3	a_3	a_3

The **unique stable matching** is $\{(a_1, d_1), (a_2, d_2), (a_3, d_3)\}$.

After admissions it turns out that **each college has exactly one bed**, both a_1 and a_2 always need a bed, while a_3 never needs one.

Applicant a_2 **does not get a bed** and drops out of the college.

Final matching: $\{(a_1, d_1, 1), (a_3, d_3, 0)\}$.

Motivating Example

There are 3 applicants, and 3 departments in 2 colleges: $c_1 = \{d_1, d_2\}$ and $c_2 = \{d_3\}$. Each department has a **unit capacity**. Preferences are:

a_1	a_2	a_3	$\{d_1$	$d_2\}$	$\{d_3\}$
$(d_1, 1)$	$(d_1, 1)$	$(d_1, 0)$	a_1	a_1	a_1
$(d_2, 1)$	$(d_2, 1)$	$(d_2, 0)$	a_2	a_2	a_2
$(d_3, 1)$	$(d_3, 1)$	$(d_3, 0)$	a_3	a_3	a_3

The **unique stable matching** is $\{(a_1, d_1), (a_2, d_2), (a_3, d_3)\}$.

After admissions it turns out that **each college has exactly one bed**, both a_1 and a_2 always need a bed, while a_3 never needs one.

Applicant a_2 **does not get housing** and drops out of the college.

Final matching: $\{(a_1, d_1, 1), (a_3, d_3, 0)\}$.

Motivating Example

There are 3 applicants, and 3 departments in 2 colleges: $c_1 = \{d_1, d_2\}$ and $c_2 = \{d_3\}$. Each department has a **unit capacity**. Preferences are:

a_1	a_2	a_3	$\{d_1$	$d_2\}$	$\{d_3\}$
$(d_1, 1)$	$(d_1, 1)$	$(d_1, 0)$	a_1	a_1	a_1
$(d_2, 1)$	$(d_2, 1)$	$(d_2, 0)$	a_2	a_2	a_2
$(d_3, 1)$	$(d_3, 1)$	$(d_3, 0)$	a_3	a_3	a_3

The **unique stable matching** is $\{(a_1, d_1), (a_2, d_2), (a_3, d_3)\}$.

After admissions it turns out that **each college has exactly one bed**, both a_1 and a_2 always need a bed, while a_3 never needs one.

Applicant a_2 **does not get housing** and drops out of the college.

Final matching: $\{(a_1, d_1, 1), (a_3, d_3, 0)\}$ is not stable!

Motivating Example

There are 3 applicants, and 3 departments in 2 colleges: $c_1 = \{d_1, d_2\}$ and $c_2 = \{d_3\}$. Each department has a **unit capacity**. Preferences are:

a_1	a_2	a_3	$\{d_1$	$d_2\}$	$\{d_3\}$
$(d_1, 1)$	$(d_1, 1)$	$(d_1, 0)$	a_1	a_1	a_1
$(d_2, 1)$	$(d_2, 1)$	$(d_2, 0)$	a_2	a_2	a_2
$(d_3, 1)$	$(d_3, 1)$	$(d_3, 0)$	a_3	a_3	a_3

The **unique stable matching** is $\{(a_1, d_1), (a_2, d_2), (a_3, d_3)\}$.

After admissions it turns out that **each college has exactly one bed**, both a_1 and a_2 always need a bed, while a_3 never needs one.

Applicant a_2 **does not get housing** and drops out of the college.

Final matching: $\{(a_1, d_1, 1), (a_3, d_3, 0)\}$ is not stable!

Roadmap

- General framework: possible applications
- College housing crisis
- **Stable matching: six types of blocking contracts**
- *Take-House-from-Applicant*-Stability
- *Not-Compromised-Request-from-One-Agent*-Stability
- Blocking domination: IP solution
- Concluding remarks

Fairness and Non-Wastefulness (Balinski and Sönmez, 1999)

The **absence of blocking pairs** for a matching can be guaranteed by the following features of this matching:

- A matching is **fair** if a higher-priority applicant can never envy the assignment of a lower-priority applicant for any department.
- A matching is **non-wasteful** if an applicant never wants an empty slot at a department that treats this applicant as acceptable.

Fairness: ((No) Housing-by-(No) Housing)-Blocking

There can be **four types** of **(i'-by-i)-blocking** contracts for a matching μ.

Suppose that a **chosen contract with(out) housing** $\in \mu$ is **blocked by** a **not chosen contract with(out) housing** $\notin \mu$. Thus, we should have:

a	a'	$\cdots$	d	$\cdots$
(d, i)	(d, i')		a	
μ_a			a'	

and

$$\big(\mu \backslash \{(a', d, i'), \mu_a\}\big) \cup (a, d, i) \text{ is feasible.}$$

if $i \leq i'$ (no additional college resource is needed).

Fairness: ((No) Housing-by-(No) Housing)-Blocking

There can be **four types** of **$(i'$-by-$i)$-blocking** contracts for a matching μ.

Suppose that a **chosen contract with(out) housing** $\in \mu$ is **blocked by** a **not chosen contract with(out) housing** $\notin \mu$. Thus, we should have:

a	a'	$\cdots$	d	$\cdots$
(d, i)	(d, i')		a	
μ_a			a'	

and

$(\mu \backslash \{(a', d, i'), \mu_a\}) \cup (a, d, i)$ is feasible.

if $i \leq i'$ (**no additional college resource** is needed).

Fairness: (No Housing-by-Housing)-Blocking

Now consider a **(NH-by-H)-blocking**: a **chosen contract without housing** $\in \mu$ is **blocked by** a **not chosen contract with housing** $\notin \mu$:

a	a'	$\cdots$	d	$\cdots$
$(d, 1)$	$(d, 0)$		a	
μ_a			a'	

and

$$\left(\mu \backslash \{(a', d, 0), \mu_a\}\right) \cup (a, d, 1) \text{ is feasible.}$$

after a gives up on μ_a, a college $c(d)$ has an **empty bed**.

Fairness: (No Housing-by-Housing)-Blocking

Now consider a **(NH-by-H)-blocking**: a **chosen contract without housing** $\in \mu$ is **blocked by** a **not chosen contract with housing** $\notin \mu$:

a	a'	$\cdots$	d	$\cdots$
$(d,1)$	$(d,0)$		a	
μ_a			a'	

and

$(\mu \backslash \{(a',d,0),\mu_a\}) \cup (a,d,1)$ is feasible.

after a gives up on μ_a, a college $c(d)$ has an **empty bed**.

Recall the Motivating Example

There are 3 applicants, and 3 departments in 2 colleges: $c_1 = \{d_1, d_2\}$ and $c_2 = \{d_3\}$. Each department has a **unit capacity**, and each college has exactly **one bed**. Preferences are:

a_1	a_2	a_3	$\{d_1$	$d_2\}$	$\{d_3\}$
$(d_1, 1)$	$(d_1, 1)$	$(d_1, 0)$	a_1	a_1	a_1
$(d_2, 1)$	$(d_2, 1)$	$(d_2, 0)$	a_2	a_2	a_2
$(d_3, 1)$	$(d_3, 1)$	$(d_3, 0)$	a_3	a_3	a_3

Final matching: $\{(a_1, d_1, 1), (a_3, d_3, 0)\}$.

- Contract $(a_2, d_3, 1)$ is (NH-by-H)-blocking.

Recall the Motivating Example

There are 3 applicants, and 3 departments in 2 colleges: $c_1 = \{d_1, d_2\}$ and $c_2 = \{d_3\}$. Each department has a **unit capacity**, and each college has exactly **one bed**. Preferences are:

a_1	a_2	a_3	$\{d_1$	$d_2\}$	$\{d_3\}$
$(d_1, 1)$	$(d_1, 1)$	$(d_1, 0)$	a_1	a_1	a_1
$(d_2, 1)$	$(d_2, 1)$	$(d_2, 0)$	a_2	a_2	a_2
$(d_3, 1)$	$(d_3, 1)$	$(d_3, 0)$	a_3	a_3	a_3

Final matching: $\{(a_1, d_1, 1), (a_3, d_3, 0)\}$ is unfair!

- Contract $(a_2, d_3, 1)$ is (NH-by-H)-blocking.

Non-Wastefulness: ($\varnothing$-by-No Housing)-Blocking

There can be **two types** of **($\varnothing$-by-i)-blocking** contracts for a matching μ.

First, consider a **($\varnothing$-by-NH)-blocking**: when an **unfilled department slot** is **blocked by** a **not chosen contract without housing** $\notin \mu$:

a	$\cdots$	d	$\cdots$
$(d, 0)$		a	
μ_a		$\varnothing$	

and

$$(\mu \backslash \mu_a) \cup (a, d, 0) \text{ is feasible.}$$

Non-Wastefulness: ($\varnothing$-by-No Housing)-Blocking

There can be **two types** of **($\varnothing$-by-i)-blocking** contracts for a matching μ.

First, consider a **($\varnothing$-by-NH)-blocking**: when an **unfilled department slot** is **blocked by** a **not chosen contract without housing** $\notin \mu$:

a	$\cdots$	d	$\cdots$
$(d, 0)$		a	
μ_a		$\varnothing$	

and

$(\mu \backslash \mu_a) \cup (a, d, 0)$ is feasible.

Non-Wastefulness: ($\varnothing$-by-Housing)-Blocking

Second, consider a **($\varnothing$-by-H)-blocking**: when an **unfilled department slot** is **blocked by** a **not chosen contract with housing** $\notin \mu$:

a	$\cdots$	d	$\cdots$
$(d, 1)$		a	
μ_a		$\varnothing$	

and

$(\mu \backslash \mu_a) \cup (a, d, 1)$ is feasible.

after a gives up on μ_a, a college $c(d)$ has an **empty bed**.

Non-Wastefulness: (∅-by-Housing)-Blocking

Second, consider a **(∅-by-H)-blocking**: when an **unfilled department slot** is **blocked by** a **not chosen contract with housing** $\notin \mu$:

a	$\cdots$	d	$\cdots$
$(d, 1)$		a	
μ_a		$\varnothing$	

and

$(\mu \backslash \mu_a) \cup (a, d, 1)$ is feasible.

after a gives up on μ_a, a college $c(d)$ has an **empty bed**.

Recall the Motivating Example

There are 3 applicants, and 3 departments in 2 colleges: $c_1 = \{d_1, d_2\}$ and $c_2 = \{d_3\}$. Each department has a **unit capacity**, and each college has exactly **one bed**. Preferences are:

a_1	a_2	a_3	$\{d_1$	$d_2\}$	$\{d_3\}$
$(d_1, 1)$	$(d_1, 1)$	$(d_1, 0)$	a_1	a_1	a_1
$(d_2, 1)$	$(d_2, 1)$	$(d_2, 0)$	a_2	a_2	a_2
$(d_3, 1)$	$(d_3, 1)$	$(d_3, 0)$	a_3	a_3	a_3

Final matching: $\{(a_1, d_1, 1), (a_3, d_3, 0)\}$ is unfair!

- Contract $(a_2, d_3, 1)$ is (NH-by-H)-blocking.
- Contract $(a_3, d_2, 0)$ is ($\varnothing$-by-NH)-blocking.

Recall the Motivating Example

There are 3 applicants, and 3 departments in 2 colleges: $c_1 = \{d_1, d_2\}$ and $c_2 = \{d_3\}$. Each department has a **unit capacity**, and each college has exactly **one bed**. Preferences are:

a_1	a_2	a_3	$\{d_1$	$d_2\}$	$\{d_3\}$
$(d_1, 1)$	$(d_1, 1)$	$(d_1, 0)$	a_1	a_1	a_1
$(d_2, 1)$	$(d_2, 1)$	$(d_2, 0)$	a_2	a_2	a_2
$(d_3, 1)$	$(d_3, 1)$	$(d_3, 0)$	a_3	a_3	a_3

Final matching: $\{(a_1, d_1, 1), (a_3, d_3, 0)\}$ is unfair and wasteful!

- Contract $(a_2, d_3, 1)$ is (NH-by-H)-blocking.
- Contract $(a_3, d_2, 0)$ is ($\varnothing$-by-NH)-blocking.

Six Types of Blocking Contracts

μ	Type	College Resource
Unfair	(NH -by- NH)	–
	(H -by- NH)	–
	(H -by- H)	–
	(NH -by- H)	✓
Wasteful	($\varnothing$ -by- NH)	–
	($\varnothing$ -by- H)	✓

Recall the Motivating Example

There are 3 applicants, and 3 departments in 2 colleges: $c_1 = \{d_1, d_2\}$ and $c_2 = \{d_3\}$. Each department has a **unit capacity**, and each college has exactly **one bed**. Preferences are:

a_1	a_2	a_3	$\{d_1$	$d_2\}$	d_3
$(d_1, 1)$	$(d_1, 1)$	$(d_1, 0)$	a_1	a_1	a_1
$(d_2, 1)$	$(d_2, 1)$	$(d_2, 0)$	a_2	a_2	a_2
$(d_3, 1)$	$(d_3, 1)$	$(d_3, 0)$	a_3	a_3	a_3

Final matching: $\{(a_1, d_1, 1), (a_3, d_3, 0)\}$ is not stable!

Stable matching: $\{(a_1, d_1, 1), (a_2, d_3, 1), (a_3, d_2, 0)\}$[2].

[2] Both blocking contracts are satisfied.

Recall the Motivating Example

There are 3 applicants, and 3 departments in 2 colleges: $c_1 = \{d_1, d_2\}$ and $c_2 = \{d_3\}$. Each department has a **unit capacity**, and each college has exactly **one bed**. Preferences are:

a_1	a_2	a_3	$\{d_1$	$d_2\}$	d_3
$(d_1, 1)$	$(d_1, 1)$	$(d_1, 0)$	a_1	a_1	a_1
$(d_2, 1)$	$(d_2, 1)$	$(d_2, 0)$	a_2	a_2	a_2
$(d_3, 1)$	$(d_3, 1)$	$(d_3, 0)$	a_3	a_3	a_3

Final matching: $\{(a_1, d_1, 1), (a_3, d_3, 0)\}$ is not stable!

Stable matching: $\{(a_1, d_1, 1), (a_2, d_3, 1), (a_3, d_2, 0)\}$[2].

[2] Both blocking contracts are satisfied.

A Stable Matching May Not Exist!

Consider the following CAH with 2 applicants and 2 departments in one college. Quotas are $q_{d_1} = q_{d_2} = q_c = 1$. Preferences are:

a_1	a_2	$\{d_1$	$d_2\}$
$(d_1, 1)$	$(d_2, 1)$	a_2	a_1
$(d_2, 1)$	$(d_1, 1)$	a_1	a_2

There is **no stable matching** under this market:

- $\{\}$ and $\{(a_2, d_1, 1)\}$ have a ($\varnothing$-by-H)-blocking contract $\{(a_2, d_2, 1)\}$;
- $\{(a_2, d_2, 1)\}$ has a (H-by-H)-blocking contract $\{(a_1, d_2, 1)\}$;
- $\{(a_1, d_2, 1)\}$ has a ($\varnothing$-by-H)-blocking contract $\{(a_1, d_1, 1)\}$;
- $\{(a_1, d_1, 1)\}$ has a (H-by-H)-blocking contract $\{(a_2, d_1, 1)\}$.

A Stable Matching May Not Exist!

Consider the following CAH with 2 applicants and 2 departments in one college. Quotas are $q_{d_1} = q_{d_2} = q_c = 1$. Preferences are:

a_1	a_2	$\{d_1$	$d_2\}$
$(d_1, 1)$	$(d_2, 1)$	a_2	a_1
$(d_2, 1)$	$(d_1, 1)$	a_1	a_2

There is **no stable matching** under this market:

- $\{\}$ and $\{(a_2, d_1, 1)\}$ have a ($\varnothing$-by-H)-blocking contract $\{(a_2, d_2, 1)\}$;
- $\{(a_2, d_2, 1)\}$ has a (H-by-H)-blocking contract $\{(a_1, d_2, 1)\}$;
- $\{(a_1, d_2, 1)\}$ has a ($\varnothing$-by-H)-blocking contract $\{(a_1, d_1, 1)\}$;
- $\{(a_1, d_1, 1)\}$ has a (H-by-H)-blocking contract $\{(a_2, d_1, 1)\}$.

A Stable Matching May Not Exist!

Consider the following CAH with 2 applicants and 2 departments in one college. Quotas are $q_{d_1} = q_{d_2} = q_c = 1$. Preferences are:

a_1	a_2	$\{d_1$	$d_2\}$
$(d_1, 1)$	$(d_2, 1)$	a_2	a_1
$(d_2, 1)$	$(d_1, 1)$	a_1	a_2

There is **no stable matching** under this market:

- $\{\}$ and $\{(a_2, d_1, 1)\}$ have a ($\varnothing$-by-H)-blocking contract $\{(a_2, d_2, 1)\}$;
- $\{(a_2, d_2, 1)\}$ has a (H-by-H)-blocking contract $\{(a_1, d_2, 1)\}$;
- $\{(a_1, d_2, 1)\}$ has a ($\varnothing$-by-H)-blocking contract $\{(a_1, d_1, 1)\}$;
- $\{(a_1, d_1, 1)\}$ has a (H-by-H)-blocking contract $\{(a_2, d_1, 1)\}$.

A Stable Matching May Not Exist!

Consider the following CAH with 2 applicants and 2 departments in one college. Quotas are $q_{d_1} = q_{d_2} = q_c = 1$. Preferences are:

a_1	a_2	$\{d_1$	$d_2\}$
$(d_1, 1)$	$(d_2, 1)$	a_2	a_1
$(d_2, 1)$	$(d_1, 1)$	a_1	a_2

There is **no stable matching** under this market:

- $\{\}$ and $\{(a_2, d_1, 1)\}$ have a ($\varnothing$-by-H)-blocking contract $\{(a_2, d_2, 1)\}$;
- $\{(a_2, d_2, 1)\}$ has a (H-by-H)-blocking contract $\{(a_1, d_2, 1)\}$;
- $\{(a_1, d_2, 1)\}$ has a ($\varnothing$-by-H)-blocking contract $\{(a_1, d_1, 1)\}$;
- $\{(a_1, d_1, 1)\}$ has a (H-by-H)-blocking contract $\{(a_2, d_1, 1)\}$.

A Stable Matching May Not Exist!

Consider the following CAH with 2 applicants and 2 departments in one college. Quotas are $q_{d_1} = q_{d_2} = q_c = 1$. Preferences are:

a_1	a_2	$\{d_1$	$d_2\}$
$(d_1, 1)$	$(d_2, 1)$	a_2	a_1
$(d_2, 1)$	$(d_1, 1)$	a_1	a_2

There is **no stable matching** under this market:

- $\{\}$ and $\{(a_2, d_1, 1)\}$ have a ($\varnothing$-by-H)-blocking contract $\{(a_2, d_2, 1)\}$;
- $\{(a_2, d_2, 1)\}$ has a (H-by-H)-blocking contract $\{(a_1, d_2, 1)\}$;
- $\{(a_1, d_2, 1)\}$ has a ($\varnothing$-by-H)-blocking contract $\{(a_1, d_1, 1)\}$;
- $\{(a_1, d_1, 1)\}$ has a (H-by-H)-blocking contract $\{(a_2, d_1, 1)\}$.

A Stable Matching May Not Exist!

Consider the following CAH with 2 applicants and 2 departments in one college. Quotas are $q_{d_1} = q_{d_2} = q_c = 1$. Preferences are:

a_1	a_2	$\{d_1$	$d_2\}$
$(d_1, 1)$	$(d_2, 1)$	a_2	a_1
$(d_2, 1)$	$(d_1, 1)$	a_1	a_2

There is **no stable matching** under this market:

- $\{\}$ and $\{(a_2, d_1, 1)\}$ have a ($\varnothing$-by-H)-blocking contract $\{(a_2, d_2, 1)\}$;
- $\{(a_2, d_2, 1)\}$ has a (H-by-H)-blocking contract $\{(a_1, d_2, 1)\}$;
- $\{(a_1, d_2, 1)\}$ has a ($\varnothing$-by-H)-blocking contract $\{(a_1, d_1, 1)\}$;
- $\{(a_1, d_1, 1)\}$ has a (H-by-H)-blocking contract $\{(a_2, d_1, 1)\}$.

Roadmap

- General framework: possible applications
- College housing crisis
- Stable matching: six types of blocking contracts
- *Take-House-from-Applicant*-Stability
- *Not-Compromised-Request-from-One-Agent*-Stability
- Blocking domination: IP solution
- Concluding remarks

Blocking Contracts under THfA-Stability

Proposition

A blocking contract is not tolerated under THfA-stability if and only if it requires an additional college resource.

THfA-st. μ	Type	College Resource
Unfair	(NH -by- NH)	–
	(H -by- NH)	–
	(H -by- H)	–
	(NH -by- H)	✓
Wasteful	($\varnothing$ -by- NH)	–
	($\varnothing$ -by- H)	✓

Not Chosen Applicant Claims an Empty Seat

Consider the following CAH with **4 applicants** and **2 colleges** containing **2 departments each**. First college quotas are $q_{d_1} = q_{d_2} = q_{c_1} = 1$. Second college quotas are $q_{d_3} = 2$, $q_{d_4} = q_{c_2} = 1$. Preferences are:

a_1	a_2	a_3	a_4	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_3, 1)$	$(d_4, 0)$	a_2	a_1	a_1	a_4
$(d_2, 1)$	$(d_1, 1)$			a_1	a_2	a_3	
$(d_3, 0)$							

Cumulative offer mechanism with Ch_d^* (**no stable matching**):

- ***Step 0***: c_1 gives bed to d_1, and c_2 gives bed to d_4.

...

- ***Final matching***: $\{(a_1, d_3, 0), (a_2, d_1, 1), (a_4, d_4, 0)\}$.

Bad news: not admitted applicant a_3 has a ($\varnothing$-by-H)-blocking contract.

Not Chosen Applicant Claims an Empty Seat

Consider the following CAH with **4 applicants** and **2 colleges** containing **2 departments each**. First college quotas are $q_{d_1} = q_{d_2} = q_{c_1} = 1$. Second college quotas are $q_{d_3} = 2$, $q_{d_4} = q_{c_2} = 1$. Preferences are:

a_1	a_2	a_3	a_4	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_3, 1)$	$(d_4, 0)$	a_2	a_1	a_1	a_4
$(d_2, 1)$	$(d_1, 1)$			a_1	a_2	a_3	
$(d_3, 0)$							

Cumulative offer mechanism with Ch^*_d (**no stable matching**):

- ***Step 0***: c_1 gives bed to d_1, and c_2 gives bed to d_4.

...

- ***Final matching***: $\{(a_1, d_3, 0), (a_2, d_1, 1), (a_4, d_4, 0)\}$.

Bad news: not admitted applicant a_3 has a ($\varnothing$-by-H)-blocking contract.

Not Chosen Applicant Claims an Empty Seat

Consider the following CAH with **4 applicants** and **2 colleges** containing **2 departments each**. First college quotas are $q_{d_1} = q_{d_2} = q_{c_1} = 1$. Second college quotas are $q_{d_3} = 2$, $q_{d_4} = q_{c_2} = 1$. Preferences are:

a_1	a_2	a_3	a_4	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_3, 1)$	$(d_4, 0)$	a_2	a_1	a_1	a_4
$(d_2, 1)$	$(d_1, 1)$			a_1	a_2	a_3	
$(d_3, 0)$							

Cumulative offer mechanism with Ch^*_d (**no stable matching**):

- ***Step 0***: c_1 gives bed to d_1, and c_2 gives bed to d_4.

...

- ***Final matching***: $\{(a_1, d_3, 0), (a_2, d_1, 1), (a_4, d_4, 0)\}$.

Bad news: not admitted applicant a_3 has a ($\varnothing$-by-H)-blocking contract.

Not Chosen Applicant Claims an Empty Seat

Consider the following CAH with **4 applicants** and **2 colleges** containing **2 departments each**. First college quotas are $q_{d_1} = q_{d_2} = q_{c_1} = 1$. Second college quotas are $q_{d_3} = 2$, $q_{d_4} = q_{c_2} = 1$. Preferences are:

a_1	a_2	a_3	a_4	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_3, 1)$	$(d_4, 0)$	a_2	a_1	a_1	a_4
$(d_2, 1)$	$(d_1, 1)$			a_1	a_2	a_3	
$(d_3, 0)$							

Cumulative offer mechanism with Ch^*_d (**no stable matching**):

- ***Step 0***: c_1 gives bed to d_1, and c_2 gives bed to d_4.

...

- ***Final matching***: $\{(a_1, d_3, 0), (a_2, d_1, 1), (a_4, d_4, 0)\}$.

Bad news: not admitted applicant a_3 has a ($\varnothing$-by-H)-blocking contract.

Roadmap

- General framework: possible applications
- College housing crisis
- Stable matching: six types of blocking contracts
- *Take-House-from-Applicant*-Stability
- ***Not-Compromised-Request-from-One-Agent*-Stability**
- Blocking domination: IP solution
- Concluding remarks

Compromised ($\varnothing$-by-H)-Blocking Contract

We call a ($\varnothing$-by-H)-blocking under μ contract $(a, d, 1)$ a **compromised blocking contract** if there exists another applicant a', s.t.

- a' has an acceptable pair $(d, 1)$ that he prefers to $\mu_{a'}$,
- d prefers a' to a, and
- $(a', d, 1)$ is not ($\varnothing$-by-H)-blocking.

Compromised Contract: Example

Recall the following CAH with 2 applicants and 2 departments in one college. Quotas are $q_{d_1} = q_{d_2} = q_c = 1$. Preferences are:

a_1	a_2	$\{d_1$	$d_2\}$
$(d_1, 1)$	$(d_2, 1)$	a_2	a_1
$(d_2, 1)$	$(d_1, 1)$	a_1	a_2

Consider the following matching: $\{(a_2, d_1, 1)\}$.

There is only **one blocking contract**: ($\varnothing$-by-H)-blocking $\{(a_2, d_2, 1)\}$.

But it is **compromised**, since there is another applicant a_1, s.t.

- $(d_2, 1)$ is acceptable for a_1,
- d_2 prefers a_1 to a_2, but
- $\{(a_2, d_1, 1), (a_1, d_2, 1)\}$ is not feasible.

Compromised Contract: Example

Recall the following CAH with 2 applicants and 2 departments in one college. Quotas are $q_{d_1} = q_{d_2} = q_c = 1$. Preferences are:

a_1	a_2	$\{d_1$	$d_2\}$
$(d_1, 1)$	$(d_2, 1)$	a_2	a_1
$(d_2, 1)$	$(d_1, 1)$	a_1	a_2

Consider the following **NC-RfOA-stable** matching: $\{(a_2, d_1, 1)\}$.

There is only **one blocking contract**: ($\varnothing$-by-H)-blocking $\{(a_2, d_2, 1)\}$.

But it is **compromised**, since there is another applicant a_1, s.t.

- $(d_2, 1)$ is acceptable for a_1,
- d_2 prefers a_1 to a_2, but
- $\{(a_2, d_1, 1), (a_1, d_2, 1)\}$ is not feasible.

Seven Types of Blocking Contracts

THfA-st. μ	Type	College Resource
Unfair	(NH -by- NH)	–
	(H -by- NH)	–
	(H -by- H)	–
	(NH -by- H)	✓
Wasteful	(∅ -by- NH)	–
	(∅ -by- H) ^{nc}	✓
	(∅ -by- H) ^{c}	✓

Not-Compromised-Request-from-One-Agent Protocol

A not stable matching μ is chosen. Once an applicant a with a blocking contract (a, d, i) decides to go for it, the following **sequence of events should happen**:

1. a gives up on μ_a (all resources from μ_a are now returned back to a corresponding college);
2. a approaches a college of interest $c(d)$ and,
   - if $i = 0$, requests a needed place at d either from $c(d)$, or from an admitted to d lower ranked applicant;
   - if $i = 1$, **requests both** a needed place at d and a bed **either from an admitted** to d lower ranked **applicant**, **or directly from college** $c(d)$, if (a, d, i) **is not compromised**.

If after this sequence applicant a **is not admitted** with (d, i) then **we tolerate this blocking contract** (a, d, i) under *Not-Compromised-Request-from-One-Agent* protocol.

Not-Compromised-Request-from-One-Agent-Stability

NC-RfOA-Stable Matching

A matching is **NC-RfOA-stable** if it is individually rational, and any blocking contract is tolerated under NC-RfOA protocol.

NC-RfOA-st. μ	Type	College Resource
Unfair	(NH -by- NH)	–
	(H -by- NH)	–
	(H -by- H)	–
	(NH -by- H)	✓
Wasteful	($\varnothing$ -by- NH)	–
	($\varnothing$ -by- H)^{nc}	✓
	($\varnothing$ -by- H)^{c}	✓

No Unfilled Seats Are Desired By Not Admitted Applicants

Proposition

If there exists a ($\varnothing$-by-H)-blocking contract $(a, d, 1)$ under a NC-RfOA-stable matching, then applicant a is already admitted to $c(d)$.

Introducing Cutoffs

For each department **align** its preferences over applicants with applicants preferences over acceptable contracts for this department.

Given the set of **cutoffs** for department preferences over contracts (each department has one **seat cutoff** that is weakly lower than one **housing cutoff**) we can always construct an *allocation*: each applicant chooses the best pair (d, i) which is above the corresponding cutoff.

a_1	a_2	$\{d_1$	$d_2\}$
$(d_1, 1)$	$(d_2, 1)$	a_2	a_1
$(d_2, 1)$	$(d_1, 1)$	a_1	a_2
	$(d_2, 0)$		

Allocation: $\{(a_1, d_2, 1), (a_2, d_2, 1)\}$

Introducing Cutoffs

For each department **align** its preferences over applicants with applicants preferences over acceptable contracts for this department.

Given the set of **cutoffs** for department preferences over contracts (each department has one **seat cutoff** that is weakly lower than one **housing cutoff**) we can always construct an *allocation*: each applicant chooses the best pair (d, i) which is above the corresponding cutoff.

a_1	a_2	$\{d_1$	$d_2\}$
$(d_1, 1)$	$(d_2, 1)$	$(a_2, 1)$	$(a_1, 1)$
$(d_2, 1)$	$(d_1, 1)$	$(a_1, 1)$	$(a_2, 1)$
	$(d_2, 0)$		$(a_2, 0)$

Allocation: $\{(a_1, d_2, 1), (a_2, d_2, 1)\}$

Introducing Cutoffs

For each department **align** its preferences over applicants with applicants preferences over acceptable contracts for this department.

Given the set of **cutoffs** for department preferences over contracts (each department has one **seat cutoff** that is weakly lower than one housing cutoff) we can always construct an *allocation*: each applicant chooses the best pair (d, i) which is above the corresponding cutoff.

a_1	a_2	$\{d_1$	$d_2\}$
$(d_1, 1)$	$(d_2, 1)$	$(a_2, 1)$	$(a_1, 1)$
$(d_2, 1)$	$(d_1, 1)$	$\underline{(a_1, 1)}$	$(a_2, 1)$
	$(d_2, 0)$		$\underline{(a_2, 0)}$

Allocation: $\{(a_1, d_2, 1), (a_2, d_2, 1)\}$

Introducing Cutoffs

For each department **align** its preferences over applicants with applicants preferences over acceptable contracts for this department.

Given the set of **cutoffs** for department preferences over contracts (each department has one **seat cutoff** that is weakly lower than one **housing cutoff**) we can always construct an *allocation*: each applicant chooses the best pair (d, i) which is above the corresponding cutoff.

a_1	a_2	$\{d_1$	$d_2\}$
$(d_1, 1)$	$(d_2, 1)$	$(a_2, 1)$	$(a_1, 1)$
$(d_2, 1)$	$(d_1, 1)$	$(a_1, 1)$	$(a_2, 1)$
	$(d_2, 0)$		$(a_2, 0)$

Allocation: $\{(a_1, d_2, 1), (a_2, d_2, 1)\}$

Introducing Cutoffs

For each department **align** its preferences over applicants with applicants preferences over acceptable contracts for this department.

Given the set of **cutoffs** for department preferences over contracts (each department has one **seat cutoff** that is weakly lower than one **housing cutoff**) we can always construct an *allocation*: each applicant chooses the best pair (d, i) which is above the corresponding cutoff.

a_1	a_2	$\{d_1$	$d_2\}$
$\cancel{(d_1, 1)}$	$(d_2, 1)$	$(a_2, 1)$	$(a_1, 1)$
$(d_2, 1)$	$(d_1, 1)$	$\cancel{(a_1, 1)}$	$(a_2, 1)$
	$(d_2, 0)$		$(a_2, 0)$

Allocation: $\{(a_1, d_2, 1), (a_2, d_2, 1)\}$

Introducing Cutoffs

For each department **align** its preferences over applicants with applicants preferences over acceptable contracts for this department.

Given the set of **cutoffs** for department preferences over contracts (each department has one **seat cutoff** that is weakly lower than one **housing cutoff**) we can always construct an *allocation*: each applicant chooses the best pair (d, i) which is above the corresponding cutoff.

a_1	a_2	$\{d_1$	$d_2\}$
$\cancel{(d_1, 1)}$	$(d_2, 1)$	$(a_2, 1)$	$(a_1, 1)$
$(d_2, 1)$	$(d_1, 1)$	$\cancel{(a_1, 1)}$	$(a_2, 1)$
	$(d_2, 0)$		$(a_2, 0)$

Allocation: $\{(a_1, d_2, 1), (a_2, d_2, 1)\}$

NC-RfOA-Stable Mechanism

1. Align department preferences over applicants with applicants preferences over contracts. Set **the highest cutoffs** (no contracts can be chosen).
2. Consider each department one by one and try to **decrease its housing cutoff**, such that the **resulting allocation is feasible**.
3. If there have been considered all departments without a cutoff update, then continue by trying to **decrease seat cutoffs** one by one, such that the **resulting allocation is feasible**.
4. Once **the allocation has changed**, go to the second step (continue trying with **housing cutoffs again**). Otherwise, if there are no more possible seat cutoff updates, terminate the procedure.

NC-RfOA-Stable Mechanism

1. Align department preferences over applicants with applicants preferences over contracts. Set **the highest cutoffs** (no contracts can be chosen).
2. Consider each department one by one and try to **decrease its housing cutoff**, such that the **resulting allocation is feasible**.
3. If there have been considered all departments without a cutoff update, then continue by trying to **decrease seat cutoffs** one by one, such that the **resulting allocation is feasible**.
4. Once **the allocation has changed**, go to the second step (continue trying with **housing cutoffs again**). Otherwise, if there are no more possible seat cutoff updates, terminate the procedure.

NC-RfOA-Stable Mechanism

1. Align department preferences over applicants with applicants preferences over contracts. Set **the highest cutoffs** (no contracts can be chosen).
2. Consider each department one by one and try to **decrease its housing cutoff**, such that the **resulting allocation is feasible**.
3. If there have been considered all departments without a cutoff update, then continue by trying to **decrease seat cutoffs** one by one, such that the **resulting allocation is feasible**.
4. Once **the allocation has changed**, go to the second step (continue trying with **housing cutoffs again**). Otherwise, if there are no more possible seat cutoff updates, terminate the procedure.

NC-RfOA-Stable Mechanism

1. Align department preferences over applicants with applicants preferences over contracts. Set **the highest cutoffs** (no contracts can be chosen).
2. Consider each department one by one and try to **decrease its housing cutoff**, such that the **resulting allocation is feasible**.
3. If there have been considered all departments without a cutoff update, then continue by trying to **decrease seat cutoffs** one by one, such that the **resulting allocation is feasible**.
4. Once **the allocation has changed**, go to the second step (continue trying with **housing cutoffs again**). Otherwise, if there are no more possible seat cutoff updates, terminate the procedure.

NC-RfOA-Stable Mechanism: Example

Consider the following CAH with 5 applicants and 2 colleges with 2 departments each. All quotas are unit quotas. Preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_4, 0)$	$(d_3, 1)$	$(d_4, 1)$	a_3	a_1	a_4	a_5
$(d_2, 1)$	$(d_1, 1)$	$(d_1, 1)$			a_2	a_2		a_3
	$(d_2, 0)$				a_1			

NC-RfOA-Stable Mechanism: Example

All quotas are unit quotas. Aligned preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1,1)$	$(d_2,1)$	$(d_4,0)$	$(d_3,1)$	$(d_4,1)$	$(a_3,1)$	$(a_1,1)$	$(a_4,1)$	$(a_5,1)$
$(d_2,1)$	$(d_1,1)$	$(d_1,1)$			$(a_2,1)$	$(a_2,1)$		$(a_3,0)$
	$(d_2,0)$				$(a_1,1)$	$(a_2,0)$		

Seat cutoffs: –
Housing cutoffs: –
Matching: $\{\}$.

Resulting NC-RfOA-stable matching:

NC-RfOA-Stable Mechanism: Example

All quotas are unit quotas. Aligned preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_4, 0)$	$(d_3, 1)$	$(d_4, 1)$	$(a_3, 1)$	$(a_1, 1)$	$(a_4, 1)$	$(a_5, 1)$
$(d_2, 1)$	$(d_1, 1)$	$(d_1, 1)$			$(a_2, 1)$	$(a_2, 1)$		$(a_3, 0)$
	$(d_2, 0)$				$(a_1, 1)$	$(a_2, 0)$		

Seat cutoffs: –
Housing cutoffs: –
Matching: $\{\}$.

Resulting NC-RfOA-stable matching:

NC-RfOA-Stable Mechanism: Example

All quotas are unit quotas. Aligned preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_4, 0)$	$(d_3, 1)$	$(d_4, 1)$	$(a_3, 1)$	$(a_1, 1)$	$(a_4, 1)$	$(a_5, 1)$
$(d_2, 1)$	$(d_1, 1)$	$(d_1, 1)$			$(a_2, 1)$	$(a_2, 1)$		$(a_3, 0)$
	$(d_2, 0)$				$(a_1, 1)$	$(a_2, 0)$		

Seat cutoffs: –
Housing cutoffs: –
Matching: $\{(a_3, d_1, 1)\}$.

Resulting NC-RfOA-stable matching:

NC-RfOA-Stable Mechanism: Example

All quotas are unit quotas. Aligned preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_4, 0)$	$(d_3, 1)$	$(d_4, 1)$	$(a_3, 1)$	$(a_1, 1)$	$(a_4, 1)$	$(a_5, 1)$
$(d_2, 1)$	$(d_1, 1)$	$(d_1, 1)$			$(a_2, 1)$	$(a_2, 1)$		$(a_3, 0)$
	$(d_2, 0)$				$(a_1, 1)$	$(a_2, 0)$		

Seat cutoffs: –
Housing cutoffs: –
Matching: $\{(a_3, d_1, 1), (a_4, d_3, 1)\}$.

Resulting NC-RfOA-stable matching:

NC-RfOA-Stable Mechanism: Example

All quotas are unit quotas. Aligned preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_4, 0)$	$(d_3, 1)$	$(d_4, 1)$	$(a_3, 1)$	$(a_1, 1)$	$(a_4, 1)$	$(a_5, 1)$
$(d_2, 1)$	$(d_1, 1)$	$(d_1, 1)$			$(a_2, 1)$	$(a_2, 1)$		$(a_3, 0)$
	$(d_2, 0)$				$(a_1, 1)$	$(a_2, 0)$		

Seat cutoffs: –

Housing cutoffs: –

Matching: $\{(a_3, d_1, 1), (a_4, d_3, 1)\}$.

Resulting NC-RfOA-stable matching:

NC-RfOA-Stable Mechanism: Example

All quotas are unit quotas. Aligned preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_4, 0)$	$(d_3, 1)$	$(d_4, 1)$	$(a_3, 1)$	$(a_1, 1)$	$(a_4, 1)$	$(a_5, 1)$
$(d_2, 1)$	$(d_1, 1)$	$(d_1, 1)$			$(a_2, 1)$	$(a_2, 1)$		$(a_3, 0)$
	$(d_2, 0)$				$(a_1, 1)$	$(a_2, 0)$		

Seat cutoffs: –
Housing cutoffs: –
Matching: $\{(a_3, d_1, 1), (a_4, d_3, 1)\}$.

Resulting NC-RfOA-stable matching:

NC-RfOA-Stable Mechanism: Example

All quotas are unit quotas. Aligned preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1,1)$	$(d_2,1)$	$(d_4,0)$	$(d_3,1)$	$(d_4,1)$	$(a_3,1)$	$(a_1,1)$	$(a_4,1)$	$(a_5,1)$
$(d_2,1)$	$(d_1,1)$	$(d_1,1)$			$(a_2,1)$	$(a_2,1)$		$(a_3,0)$
	$(d_2,0)$				$(a_1,1)$	$(a_2,0)$		

Seat cutoffs: –
Housing cutoffs: –
Matching: $\{(a_3,d_1,1),(a_4,d_3,1)\}$.

Resulting NC-RfOA-stable matching:

NC-RfOA-Stable Mechanism: Example

All quotas are unit quotas. Aligned preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1,1)$	$(d_2,1)$	$(d_4,0)$	$(d_3,1)$	$(d_4,1)$	$(a_3,1)$	$(a_1,1)$	$(a_4,1)$	$(a_5,1)$
$(d_2,1)$	$(d_1,1)$	$(d_1,1)$			$(a_2,1)$	$(a_2,1)$		$(a_3,0)$
	$(d_2,0)$				$(a_1,1)$	$(a_2,0)$		

Seat cutoffs: –
Housing cutoffs: –
Matching: $\{(a_3,d_1,1),(a_4,d_3,1)\}$.

Resulting NC-RfOA-stable matching:

NC-RfOA-Stable Mechanism: Example

All quotas are unit quotas. Aligned preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_4, 0)$	$(d_3, 1)$	$(d_4, 1)$	$(a_3, 1)$	$(a_1, 1)$	$(a_4, 1)$	$(a_5, 1)$
$(d_2, 1)$	$(d_1, 1)$	$(d_1, 1)$			$(a_2, 1)$	$(a_2, 1)$		$(a_3, 0)$
	$(d_2, 0)$				$(a_1, 1)$	$(a_2, 0)$		

Seat cutoffs: –
Housing cutoffs: –
Matching: $\{(a_3, d_1, 1), (a_4, d_3, 1)\}$.

Resulting NC-RfOA-stable matching:

NC-RfOA-Stable Mechanism: Example

All quotas are unit quotas. Aligned preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_4, 0)$	$(d_3, 1)$	$(d_4, 1)$	$(a_3, 1)$	$(a_1, 1)$	$(a_4, 1)$	$(a_5, 1)$
$(d_2, 1)$	$(d_1, 1)$	$(d_1, 1)$			$(a_2, 1)$	$(a_2, 1)$		$(a_3, 0)$
	$(d_2, 0)$				$(a_1, 1)$	$(a_2, 0)$		

Seat cutoffs: –
Housing cutoffs: –
Matching: $\{(a_3, d_4, 0), (a_4, d_3, 1)\}$.

Resulting NC-RfOA-stable matching:

NC-RfOA-Stable Mechanism: Example

All quotas are unit quotas. Aligned preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_4, 0)$	$(d_3, 1)$	$(d_4, 1)$	$(a_3, 1)$	$(a_1, 1)$	$(a_4, 1)$	$(a_5, 1)$
$(d_2, 1)$	$(d_1, 1)$	$(d_1, 1)$			$(a_2, 1)$	$(a_2, 1)$		$(a_3, 0)$
	$(d_2, 0)$				$(a_1, 1)$	$(a_2, 0)$		

Seat cutoffs: –
Housing cutoffs: –
Matching: $\{(a_3, d_4, 0), (a_4, d_3, 1), (a_2, d_1, 1)\}$.

Resulting NC-RfOA-stable matching:

NC-RfOA-Stable Mechanism: Example

All quotas are unit quotas. Aligned preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_4, 0)$	$(d_3, 1)$	$(d_4, 1)$	$(a_3, 1)$	$(a_1, 1)$	$(a_4, 1)$	$(a_5, 1)$
$(d_2, 1)$	$(d_1, 1)$	$(d_1, 1)$			$(a_2, 1)$	$(a_2, 1)$		$(a_3, 0)$
	$(d_2, 0)$				$(a_1, 1)$	$(a_2, 0)$		

Seat cutoffs: –

Housing cutoffs: –

Matching: $\{(a_3, d_4, 0), (a_4, d_3, 1), (a_2, d_1, 1)\}$.

Resulting NC-RfOA-stable matching:

NC-RfOA-Stable Mechanism: Example

All quotas are unit quotas. Aligned preferences are:

a_1	a_2	a_3	a_4	a_5	$\{d_1$	$d_2\}$	$\{d_3$	$d_4\}$
$(d_1, 1)$	$(d_2, 1)$	$(d_4, 0)$	$(d_3, 1)$	$(d_4, 1)$	$(a_3, 1)$	$(a_1, 1)$	$\underline{(a_4, 1)}$	$(a_5, 1)$
$(d_2, 1)$	$(d_1, 1)$	$(d_1, 1)$			$\underline{(a_2, 1)}$	$(a_2, 1)$		$\underline{(a_3, 0)}$
	$(d_2, 0)$				$\underline{(a_1, 1)}$	$\underline{(a_2, 0)}$		

Seat cutoffs: –

Housing cutoffs: –

Matching: $\{(a_3, d_4, 0), (a_4, d_3, 1), (a_2, d_1, 1)\}$.

Resulting NC-RfOA-stable matching: $\{(a_3, d_4, 0), (a_4, d_3, 1), (a_2, d_1, 1)\}$.

NC-RfOA-Stable Mechanism: Results

Proposition

Cutoff Minimizing mechanism is NC-RfOA-stable, and not strategy-proof.

Corollary

NC-RfOA-stable matching always exists.

Proposition

Cutoff Minimizing mechanism may not find a stable matching if it exists.

Roadmap

- General framework: possible applications
- College housing crisis
- Stable matching: six types of blocking contracts
- *Take-House-from-Applicant*-Stability
- *Not-Compromised-Request-from-One-Agent*-Stability
- **Blocking domination: IP solution**
- Concluding remarks

Blocking Domination

- Consider two matchings μ and μ'. For each department d take the best applicant with a blocking contract a_d under μ and the best applicant with a blocking contract a'_d under μ' (if any, otherwise, set $a_d = \varnothing$ or $a'_d = \varnothing$). We say that μ **blocking-dominates** μ' if $a'_d \; R_d \; a_d$ for all $d \in D$, and $a'_{d'} \; P_{d'} \; a_{d'}$ for some $d' \in D$.

d_1	d_2
$\vdots$	$\vdots$
a_1	a_2
$\vdots$	$\vdots$
a_3	a_4

μ blocking-dominates μ'

Blocking Domination

- Consider two matchings μ and μ'. For each department d take the best applicant with a blocking contract a_d under μ and the best applicant with a blocking contract a'_d under μ' (if any, otherwise, set $a_d = \varnothing$ or $a'_d = \varnothing$). We say that μ **blocking-dominates** μ' if $a'_d \; R_d \; a_d$ for all $d \in D$, and $a'_{d'} \; P_{d'} \; a_{d'}$ for some $d' \in D$.

d_1	d_2
$\vdots$	$\vdots$
a_1	a_2
$\vdots$	$\vdots$
a_3	a_4

μ blocking-dominates μ'

Blocking Domination

- Consider two matchings μ and μ'. For each department d take the best applicant with a blocking contract a_d under μ and the best applicant with a blocking contract a'_d under μ' (if any, otherwise, set $a_d = \varnothing$ or $a'_d = \varnothing$). We say that μ **blocking-dominates** μ' if $a'_d \; R_d \; a_d$ for all $d \in D$, and $a'_{d'} \; P_{d'} \; a_{d'}$ for some $d' \in D$.

d_1	d_2
$\vdots$	$\vdots$
a_1	a_2
$\vdots$	$\vdots$
a_3	a_4

μ blocking-dominates μ'

Blocking Domination

- Consider two matchings μ and μ'. For each department d take the best applicant with a blocking contract a_d under μ and the best applicant with a blocking contract a'_d under μ' (if any, otherwise, set $a_d = \varnothing$ or $a'_d = \varnothing$). We say that μ **blocking-dominates** μ' if $a'_d \; R_d \; a_d$ for all $d \in D$, and $a'_{d'} \; P_{d'} \; a_{d'}$ for some $d' \in D$.

d_1	d_2
$\vdots$	$\vdots$
a_1	a_2
$\vdots$	$\vdots$
a_3	a_4

μ blocking-dominates μ'

Blocking Domination

- Consider two matchings μ and μ'. For each department d take the best applicant with a blocking contract a_d under μ and the best applicant with a blocking contract a'_d under μ' (if any, otherwise, set $a_d = \varnothing$ or $a'_d = \varnothing$). We say that μ **blocking-dominates** μ' if $a'_d \; R_d \; a_d$ for all $d \in D$, and $a'_{d'} \; P_{d'} \; a_{d'}$ for some $d' \in D$.

d_1	d_2
$\vdots$	$\vdots$
a_1	a_2
$\vdots$	$\vdots$
a_3	a_4

μ blocking-dominates μ'

Blocking Domination

- Consider two matchings μ and μ'. For each department d take the best applicant with a blocking contract a_d under μ and the best applicant with a blocking contract a'_d under μ' (if any, otherwise, set $a_d = \varnothing$ or $a'_d = \varnothing$). We say that μ **blocking-dominates** μ' if $a'_d \; R_d \; a_d$ for all $d \in D$, and $a'_{d'} \; P_{d'} \; a_{d'}$ for some $d' \in D$.

d_1	d_2
$\vdots$	$\vdots$
a_1	a_2
$\vdots$	$\vdots$
a_3	a_4

μ' is not comparable to μ''

Blocking Domination

- Consider two matchings μ and μ'. For each department d take the best applicant with a blocking contract a_d under μ and the best applicant with a blocking contract a'_d under μ' (if any, otherwise, set $a_d = \varnothing$ or $a'_d = \varnothing$). We say that μ **blocking-dominates** μ' if $a'_d \; R_d \; a_d$ for all $d \in D$, and $a'_{d'} \; P_{d'} \; a_{d'}$ for some $d' \in D$.

d_1	d_2
$\vdots$	$\vdots$
a_1	a_2
$\vdots$	$\vdots$
a_3	a_4

μ' is not comparable to μ''

Blocking Domination

- Consider two matchings μ and μ'. For each department d take the best applicant with a blocking contract a_d under μ and the best applicant with a blocking contract a'_d under μ' (if any, otherwise, set $a_d = \varnothing$ or $a'_d = \varnothing$). We say that μ **blocking-dominates** μ' if $a'_d \; R_d \; a_d$ for all $d \in D$, and $a'_{d'} \; P_{d'} \; a_{d'}$ for some $d' \in D$.

d_1	d_2
$\vdots$	$\vdots$
a_1	a_2
$\vdots$	$\vdots$
a_3	a_4

μ' is not comparable to μ''

Trimmed Sub-Market

- Take a CAH Δ, and for each department align its preferences over applicants with applicants preferences over acceptable contracts.
- Induced department preferences over contracts under trimmed sub-market of Δ:

d_1	d_2	d_3
$(a_1, 1)$	$(a_2, 0)$	$(a_4, 1)$
$(a_2, 0)$	$(a_4, 1)$	$(a_3, 1)$
$(a_3, 1)$	$(a_1, 1)$	$(a_2, 0)$
$(a_3, 0)$	$(a_1, 0)$	$(a_1, 1)$
$(a_4, 1)$	$(a_3, 1)$	$(a_1, 0)$

Trimmed Sub-Market

- Take a CAH Δ, and for each department align its preferences over applicants with applicants preferences over acceptable contracts.
- Induced department preferences over contracts under trimmed sub-**market** of Δ:

d_1	d_2	d_3
$(a_1, 1)$	$(a_2, 0)$	$(a_4, 1)$
$(a_2, 0)$	$(a_4, 1)$	$(a_3, 1)$
$(a_3, 1)$	$(a_1, 1)$	$(a_2, 0)$
$(a_3, 0)$	$(a_1, 0)$	$(a_1, 1)$
$(a_4, 1)$	$(a_3, 1)$	$(a_1, 0)$

Trimmed Sub-Market

- Take a CAH Δ, and for each department align its preferences over applicants with applicants preferences over acceptable contracts.
- Induced department preferences over contracts under trimmed sub-**market** of Δ:

d_1	d_2	d_3
$(a_1, 1)$	$(a_2, 0)$	$(a_4, 1)$
$(a_2, 0)$	$(a_4, 1)$	$(a_3, 1)$
$(a_3, 1)$	$(a_1, 1)$	$(a_2, 0)$
$(a_3, 0)$	$(a_1, 0)$	$(a_1, 1)$
$(a_4, 1)$	$(a_3, 1)$	$(a_1, 0)$

Trimmed Sub-Market

- Take a CAH Δ, and for each department align its preferences over applicants with applicants preferences over acceptable contracts.
- Induced department preferences over contracts under trimmed sub-**market** of Δ (setting trim thresholds):

d_1	d_2	d_3
$(a_1, 1)$	$(a_2, 0)$	$(a_4, 1)$
$(a_2, 0)$	$(a_4, 1)$	$(a_3, 1)$
$(a_3, 1)$	$(a_1, 1)$	$(a_2, 0)$
$(a_3, 0)$	$(a_1, 0)$	$(a_1, 1)$
$(a_4, 1)$	$(a_3, 1)$	$(a_1, 0)$

Trimmed Sub-Market

- Take a CAH Δ, and for each department align its preferences over applicants with applicants preferences over acceptable contracts.
- Induced department preferences over contracts under **trimmed sub-market of Δ:**

d_1	d_2	d_3
$(a_1, 1)$	$(a_2, 0)$	$(a_4, 1)$
$(a_2, 0)$		$(a_3, 1)$
		$(a_2, 0)$
$(a_3, 0)$	$(a_1, 0)$	$(a_1, 1)$
		$(a_1, 0)$

Blocking-Undominated NC-RfOA-Stable Matching: IP

1. Take a CAH Δ, **align departments' preferences** over applicants with applicants preferences over acceptable contracts.
2. **Minimize the trim thresholds** such that there still exists a stable matching, s.t. under Δ: if an applicant claims an empty seat at some department d, then she is for sure admitted to college $c(d)$.
3. **Take the resulting trimmed sub-market** Δ'.
4. **Find a student-undominated stable matching under** Δ'.

Theorem

IP procedure produces a blocking-undominated NC-RfOA-stable matching. Moreover, it produces an undominated stable matching if there exists one.

Blocking-Undominated NC-RfOA-Stable Matching: IP

1. Take a CAH Δ, **align departments' preferences** over applicants with applicants preferences over acceptable contracts.
2. **Minimize the trim thresholds** such that there still exists a stable matching, s.t. under Δ: if an applicant claims an empty seat at some department d, then she is for sure admitted to college $c(d)$.
3. Take the resulting trimmed sub-market Δ'.
4. Find a student-undominated stable matching under Δ'.

Theorem

IP procedure produces a blocking-undominated NC-RfOA-stable matching. Moreover, it produces an undominated stable matching if there exists one.

Blocking-Undominated NC-RfOA-Stable Matching: IP

1. Take a CAH Δ, **align departments' preferences** over applicants with applicants preferences over acceptable contracts.
2. **Minimize the trim thresholds** such that there still exists a stable matching, s.t. under Δ: if an applicant claims an empty seat at some department d, then she is for sure admitted to college $c(d)$.
3. **Take the resulting trimmed sub-market Δ'.**
4. Find a student-undominated stable matching under Δ'.

Theorem

IP procedure produces a blocking-undominated NC-RfOA-stable matching. Moreover, it produces an undominated stable matching if there exists one.

Blocking-Undominated NC-RfOA-Stable Matching: IP

1. Take a CAH Δ, **align departments' preferences** over applicants with applicants preferences over acceptable contracts.
2. **Minimize the trim thresholds** such that there still exists a stable matching, s.t. under Δ: if an applicant claims an empty seat at some department d, then she is for sure admitted to college $c(d)$.
3. **Take the resulting trimmed sub-market Δ'.**
4. **Find a student-undominated stable matching under Δ'.**

Theorem

IP procedure produces a blocking-undominated NC-RfOA-stable matching. Moreover, it produces an undominated stable matching if there exists one.

Blocking-Undominated NC-RfOA-Stable Matching: IP

1. Take a CAH Δ, **align departments' preferences** over applicants with applicants preferences over acceptable contracts.
2. **Minimize the trim thresholds** such that there still exists a stable matching, s.t. under Δ: if an applicant claims an empty seat at some department d, then she is for sure admitted to college $c(d)$.
3. **Take the resulting trimmed sub-market Δ'.**
4. **Find a student-undominated stable matching under Δ'.**

Theorem

IP procedure produces a blocking-undominated NC-RfOA-stable matching. Moreover, it produces an undominated stable matching if there exists one.

Roadmap

- General framework: possible applications
- College housing crisis
- Stable matching: six types of blocking contracts
- *Take-House-from-Applicant*-Stability
- *Not-Compromised-Request-from-One-Agent*-Stability
- Blocking domination: IP solution
- **Concluding remarks**

Concluding Remarks

- This paper considers a brand-new matching problem with **many applications** and focuses on one of them: **college admissions with housing quotas**.
- It proposes **two relaxations of stability** that restore existence:
  - *Take-House-from-Applicant*-stability with a strategy-proof **Cumulative Offer** process with Ch_d^*, and
  - stronger *Not-Compromised-Request-from-One-Agent*-stability with **Cutoff Minimizing** mechanism.
- Cutoff Minimizing mechanism **may not find an existing stable matching**. Thus, I propose a new notion of **blocking-undominated matching** and construct an **IP solution** that always yields a blocking-undominated NC-RfOA-stable matching.
- **Future research**: more types of contracts; overlapping regions; other ways to extend THfA-stability.