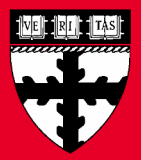

Harvard John A. Paulson
School of Engineering
and Applied Sciences

Platform Equilibrium: analyzing social welfare in online marketplaces

Gary Qiurui Ma

CIRM | 2023

Joint work with

Alon Eden

Hebrew University of Jerusalem

David Parkes

Harvard


艺得轩
徽衣轩
鹏艺园
山东中路
7:00 - 10:00
16:00 - 19:00


Miller St
Pacific Hwy
PLEASE
STAY
AT HOME


SORRY
WE'RE
CLOSED
DUE TO
COVID-19


CLOSED
Due to State
Regulations
over COVID-19

PRESS RELEASE | JUNE 8, 2020

COVID-19 to Plunge Global Economy into Worst Recession since World War II

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Are We Heading for a Historic Economic Collapse? Why the U.S. GDP Could Fall by 40%.

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BUSINESS • COVID-19

As COVID-19 Crashes the Economy, Workers and Business Owners Wonder If Anything Can Save Them From Financial Ruin

ON'S

Magazine Data Advisor Penta

T

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US:DJIA

Dow Jones

US:DJIA

Dow Jones

US:AMZN

Amazon

US:DJIA

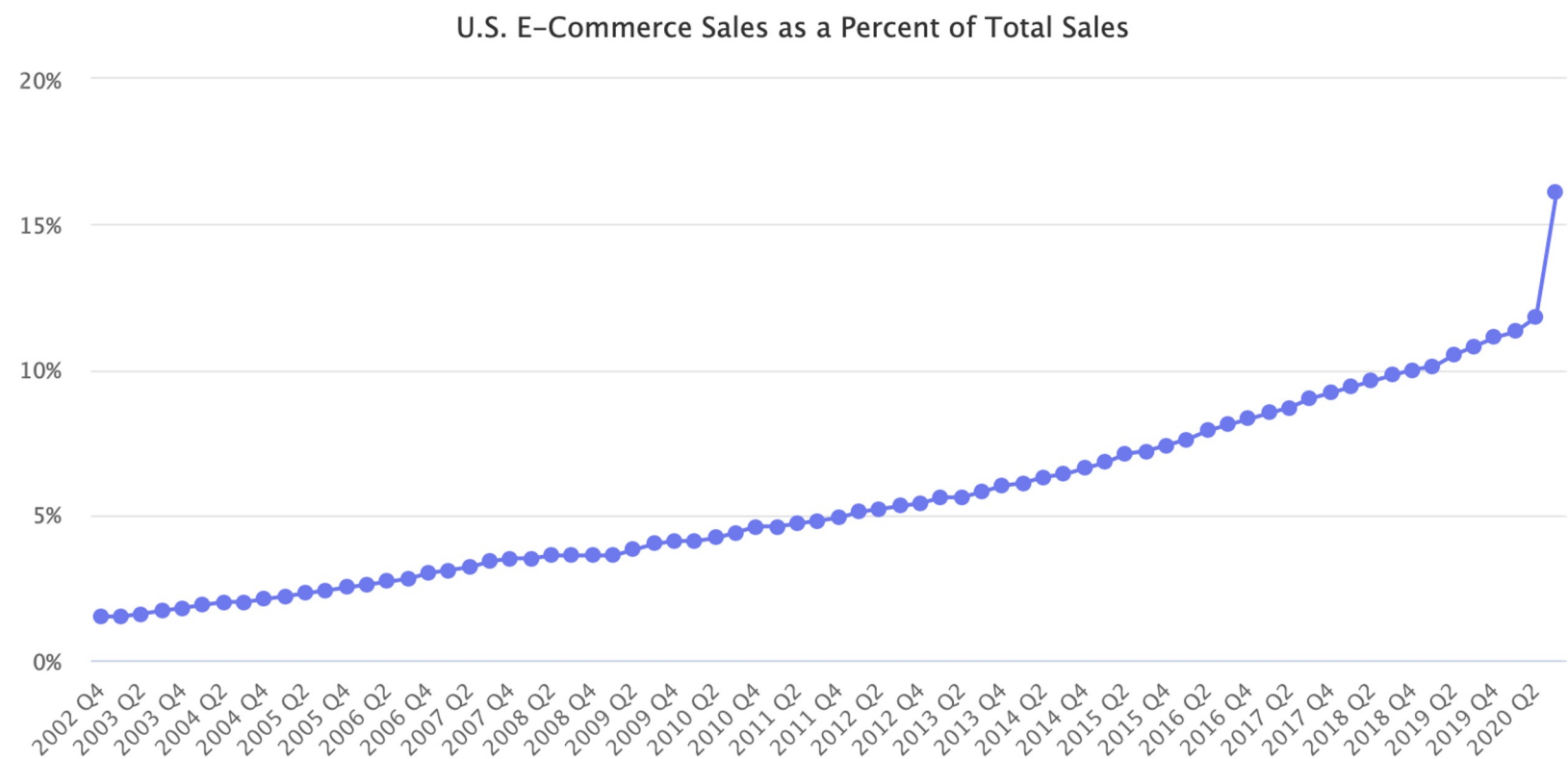
Dow Jones

US:AMZN

Amazon

PRESS RELEASE | JUNE 29, 2022

COVID-19 Drives Global Surge in use of Digital Payments



Platform based economy

amazon

Uber Eats

Platform based economy

McKinsey & Company

“restaurants’ traditional profit margins of 7 to 22 percent make covering the platforms’ delivery commissions unsustainable as delivery orders become a larger part of a restaurant’s business”

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Delivery companies are fighting city commission caps. Does anybody win?

Uber Eats

"Jersey City capped delivery app fees charged to restaurants at 10%. The next day, Uber Eats added a $3 delivery fee to local orders"

Platform based economy

To what extent do revenue-maximizing platforms enhance market efficiency?

amazon

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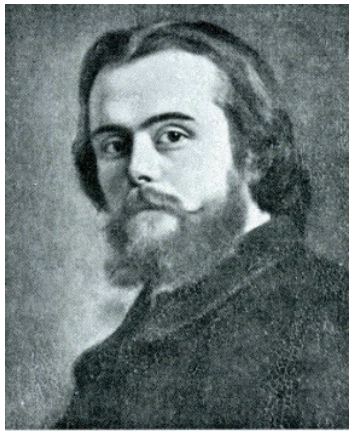
How regulation helps improve market efficiency?

Uber Eats

Walrasian Equilibrium

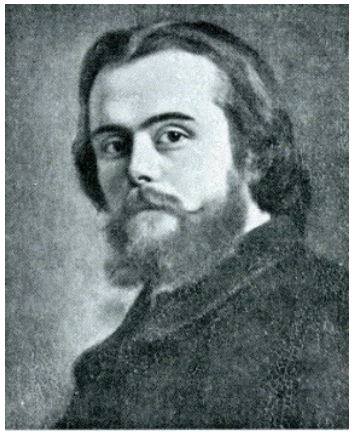


(1) allocation $\mathbf{a} = (a_1, \ldots, a_n)$
(2) item prices $\mathbf{p} = (p_1, \ldots, p_m)$

Such that

1. For buyer $\forall i, a_i$ maximizes i's utility
2. Unallocated items have 0 price

Walrasian Equilibrium



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First Welfare Theorem: Walrasian allocations maximize SW

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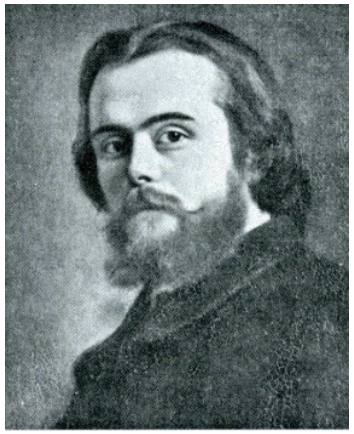

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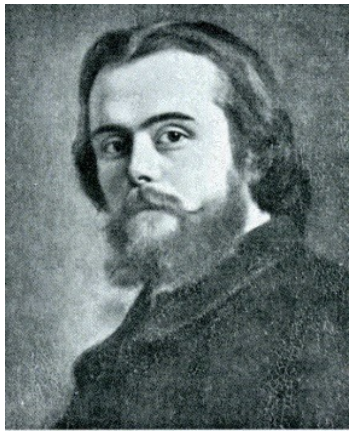

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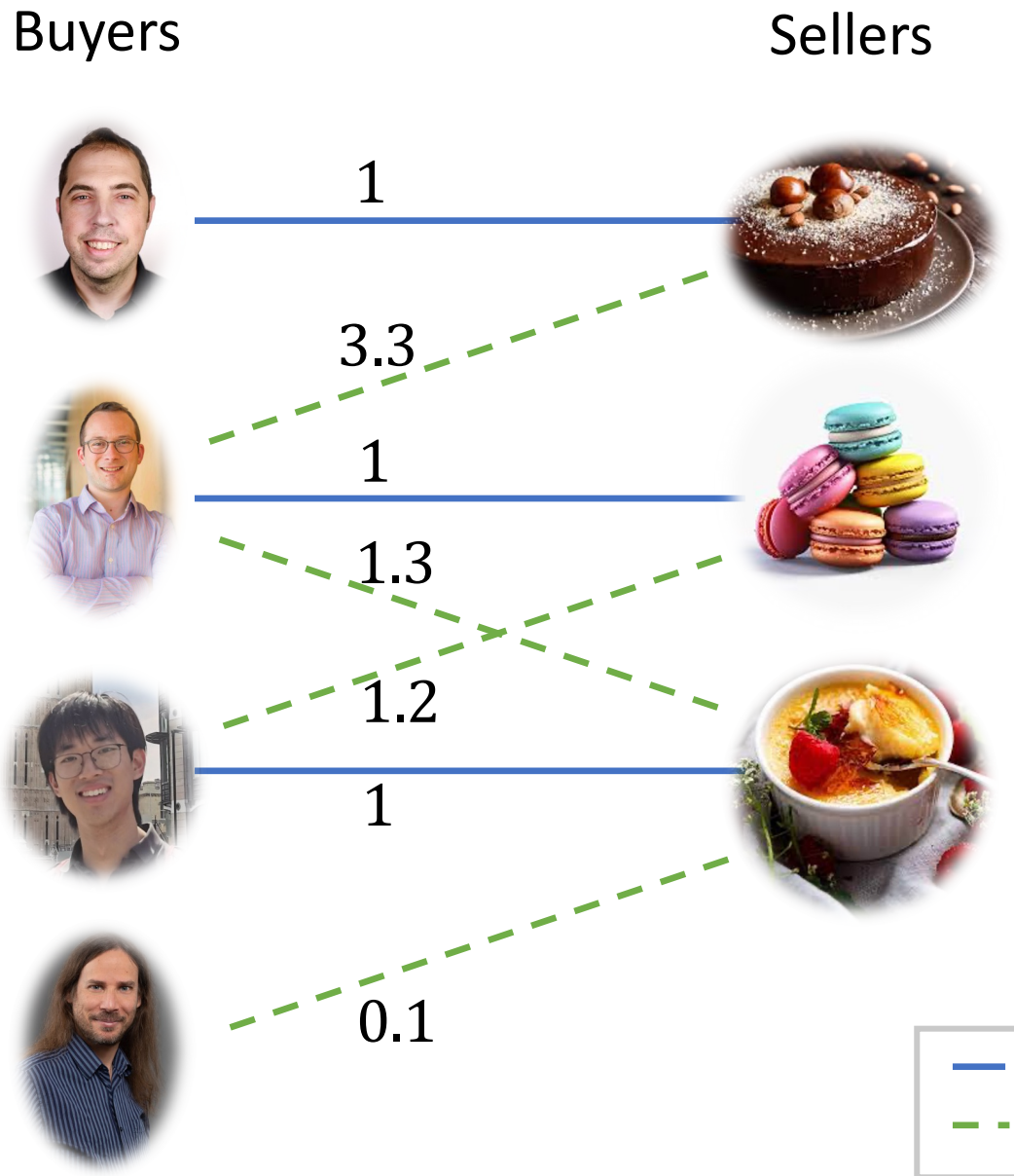
j's contribution to welfare

Platform Game

Unit-demand buyers

Unit-supply sellers



Platform Game

Unit-demand buyers

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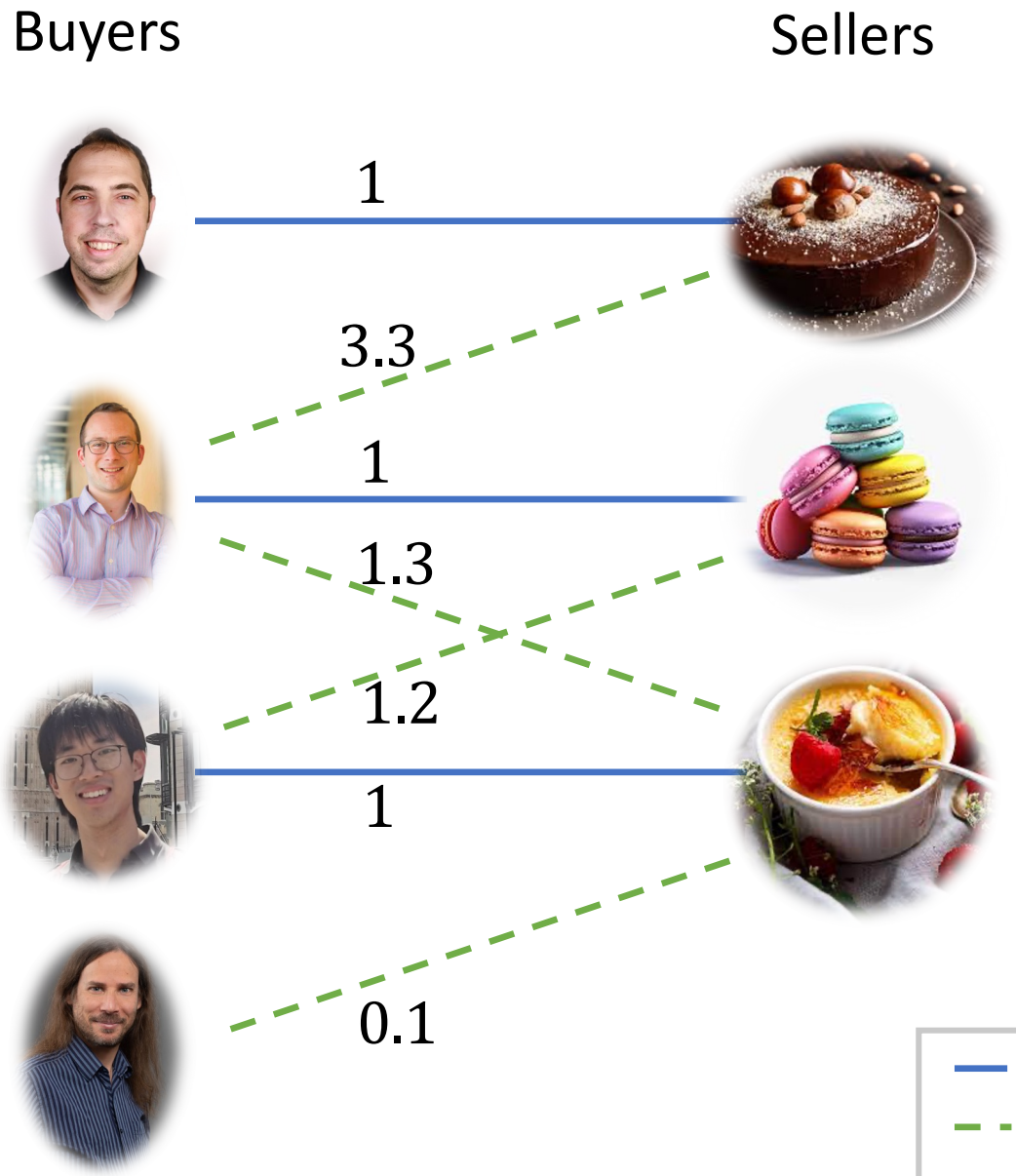
Without a platform

Transactions via active links



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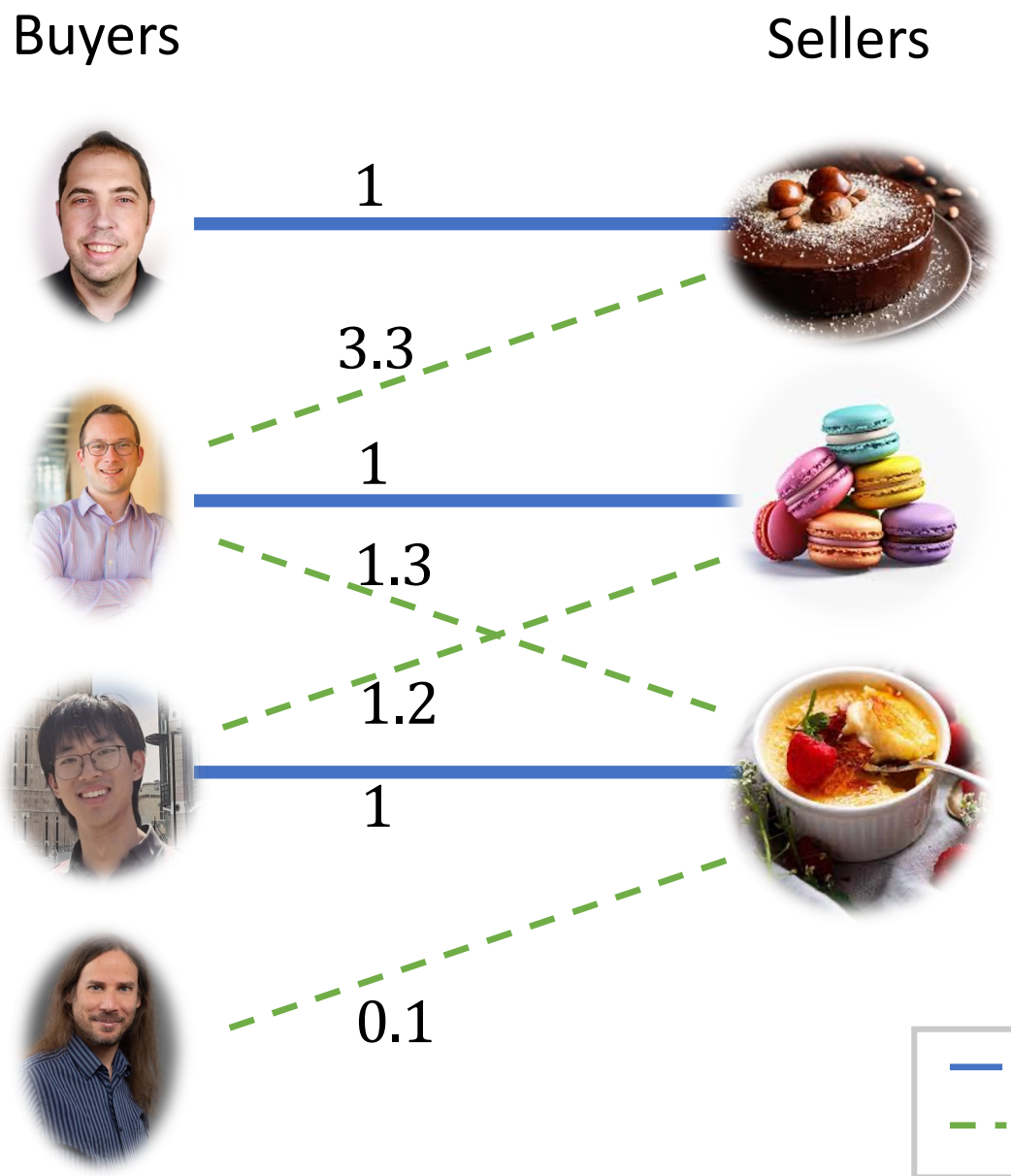

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Without a platform

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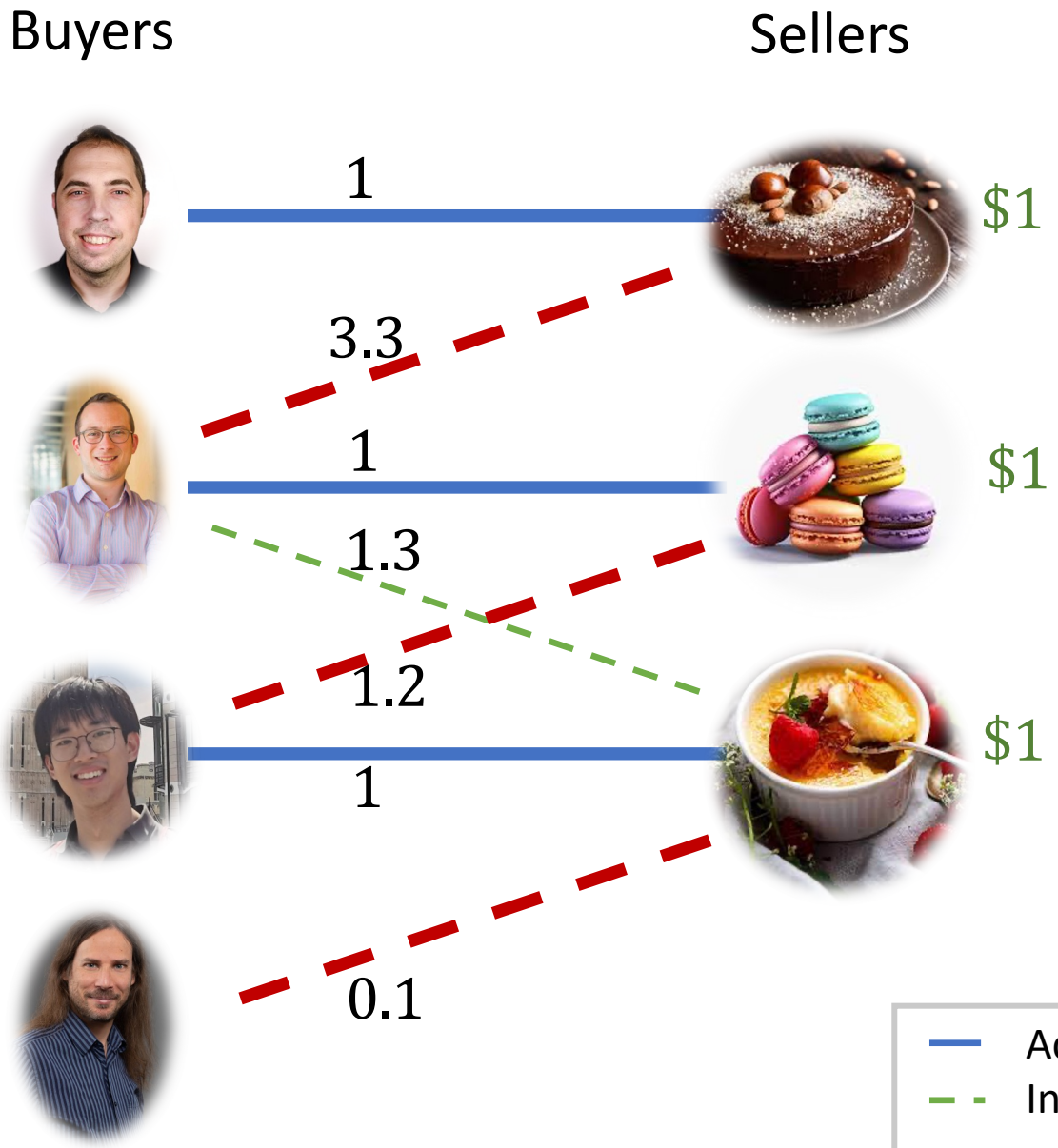
Walrasian Equilibrium (WE) is formed

Might be (very) inefficient



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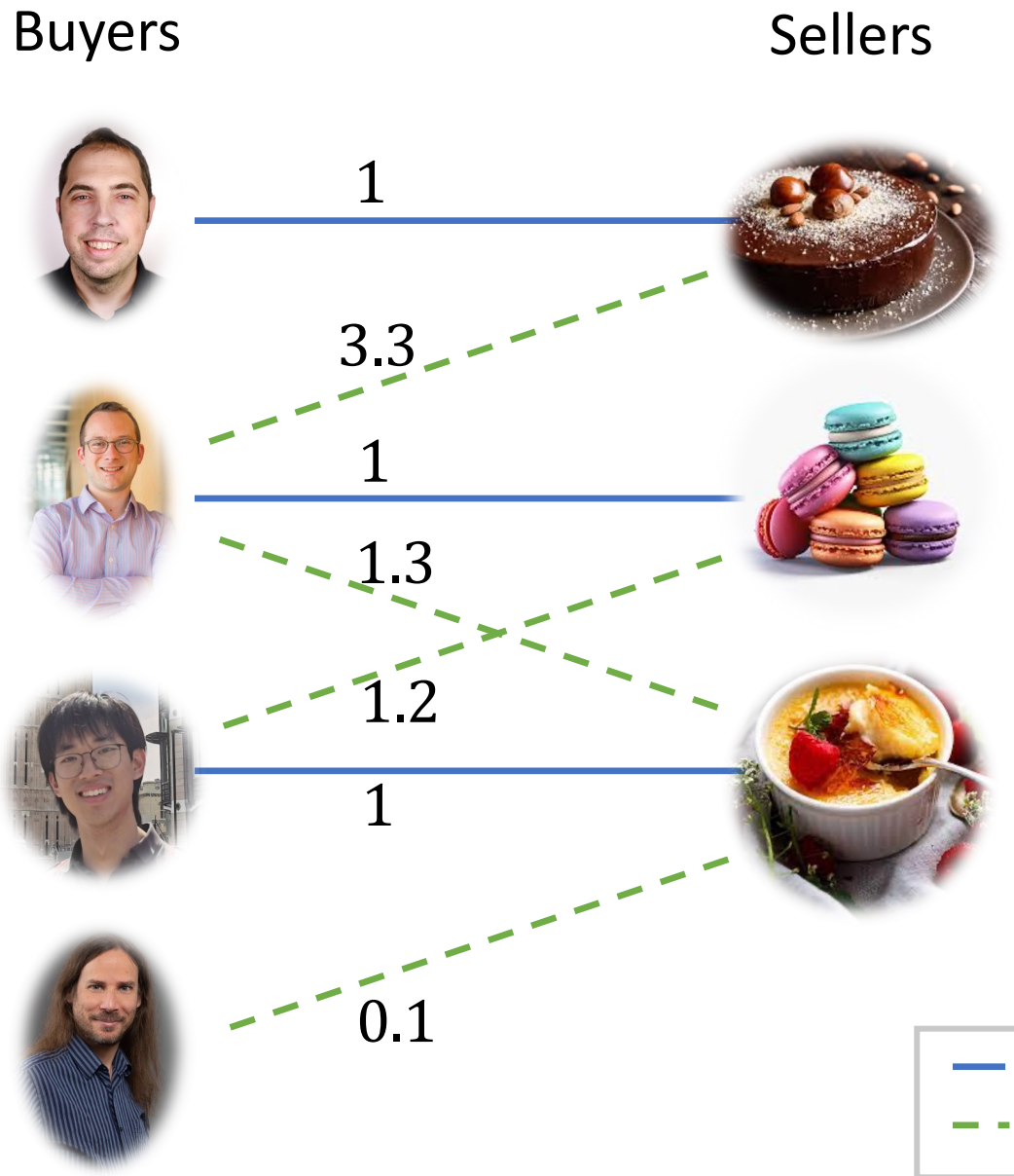

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Joining sellers pay $\alpha \cdot p_i$

Platform Game (chronological)

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1. Market is given: sellers, buyers, valuations, active/inactive links

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What's the efficiency gain?

Efficiency Gain For a Selfish Platform

Result 1: In homogenous goods markets,

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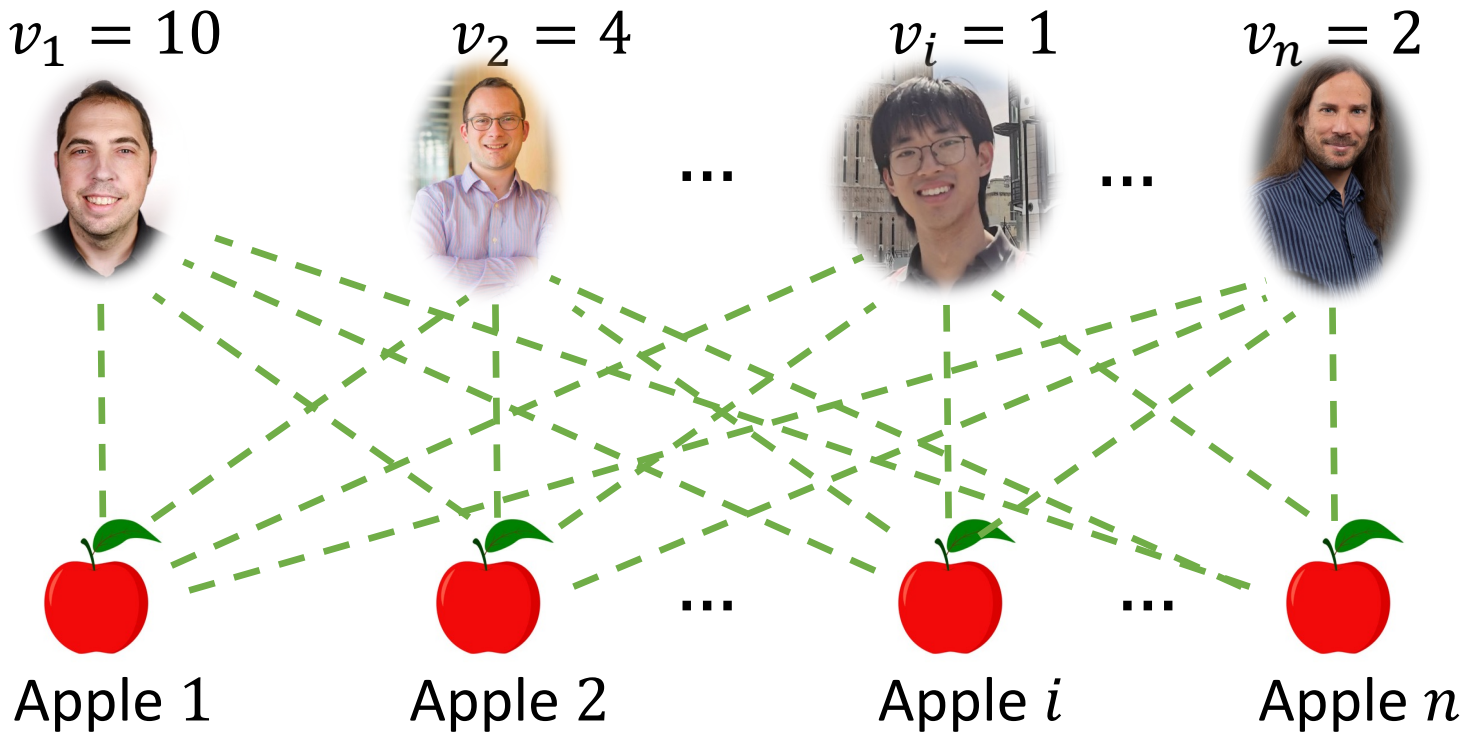

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Platforms	Amazon	UberEats	DoorDash	Grubhub
Commission Rate	8%-17%	15%-30%	15%-30%	15%-25%

Table 1. Platforms and their commission rate in the US from 2021-2022.[3]

Re1: Pure Eq. Doesn't Always Exist

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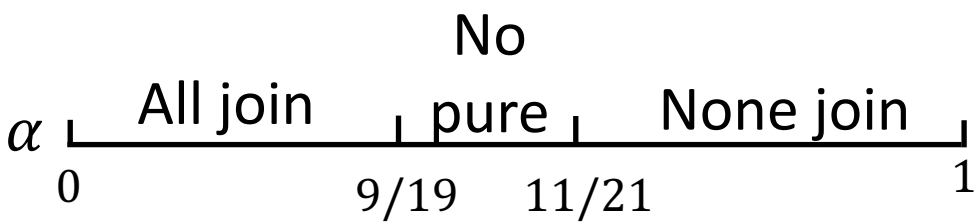

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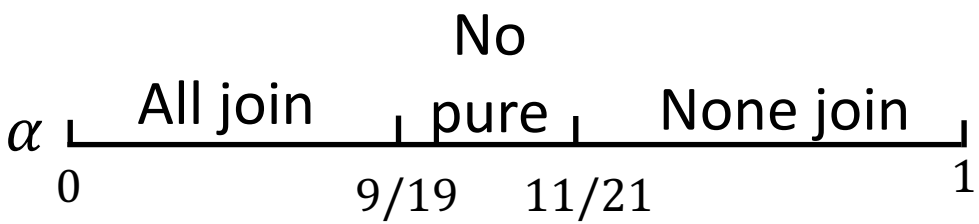

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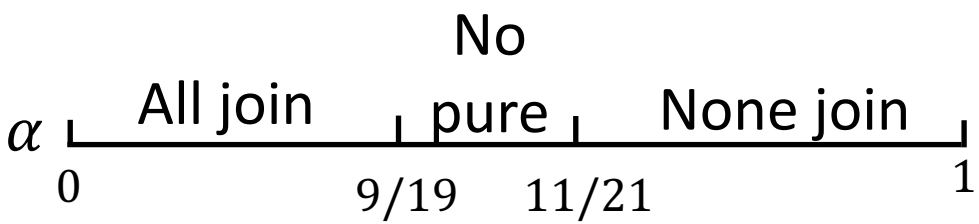

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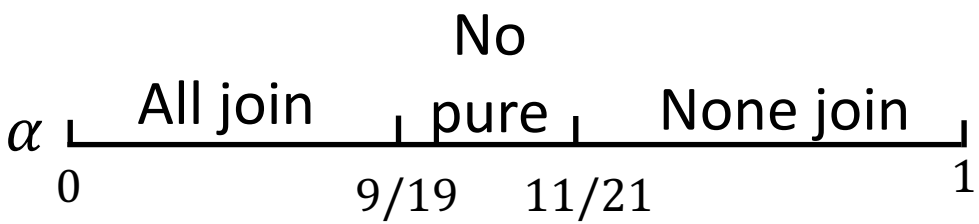

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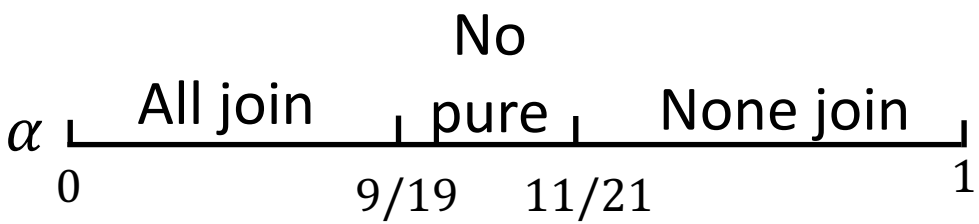

Re1: Algo for Pure Eq for homo. goods

Thm. For homo. goods, as α is lowered, sellers join one by one, forming pure equilibria.

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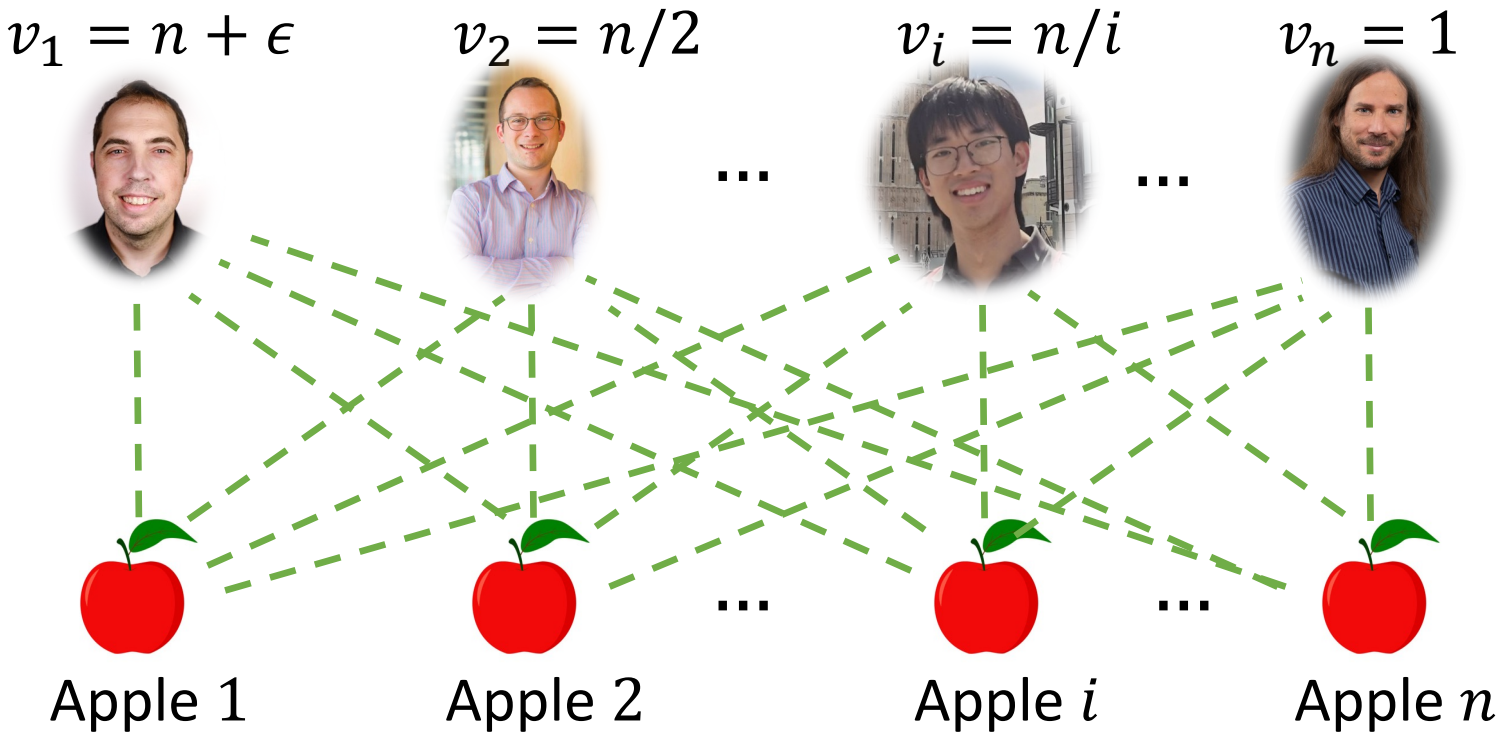
Joined sellers won't drop off

Our results also extend to mixed Eq.

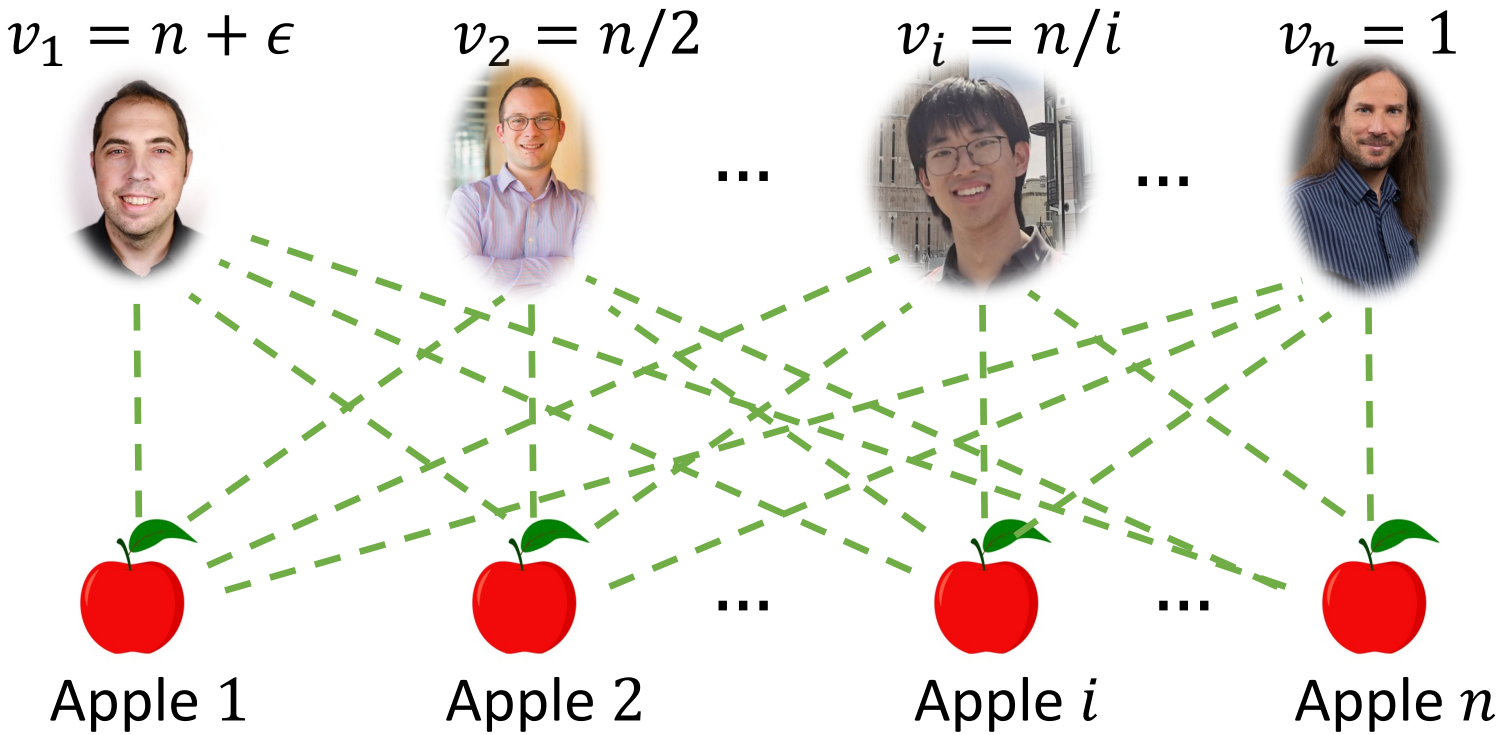
Re2: $\Omega(\log n)$ lower bound for homo. goods



Re2: $\Omega(\log n)$ lower bound for homo. goods

$\alpha^* = 1$

(sellers have 0 revenue)



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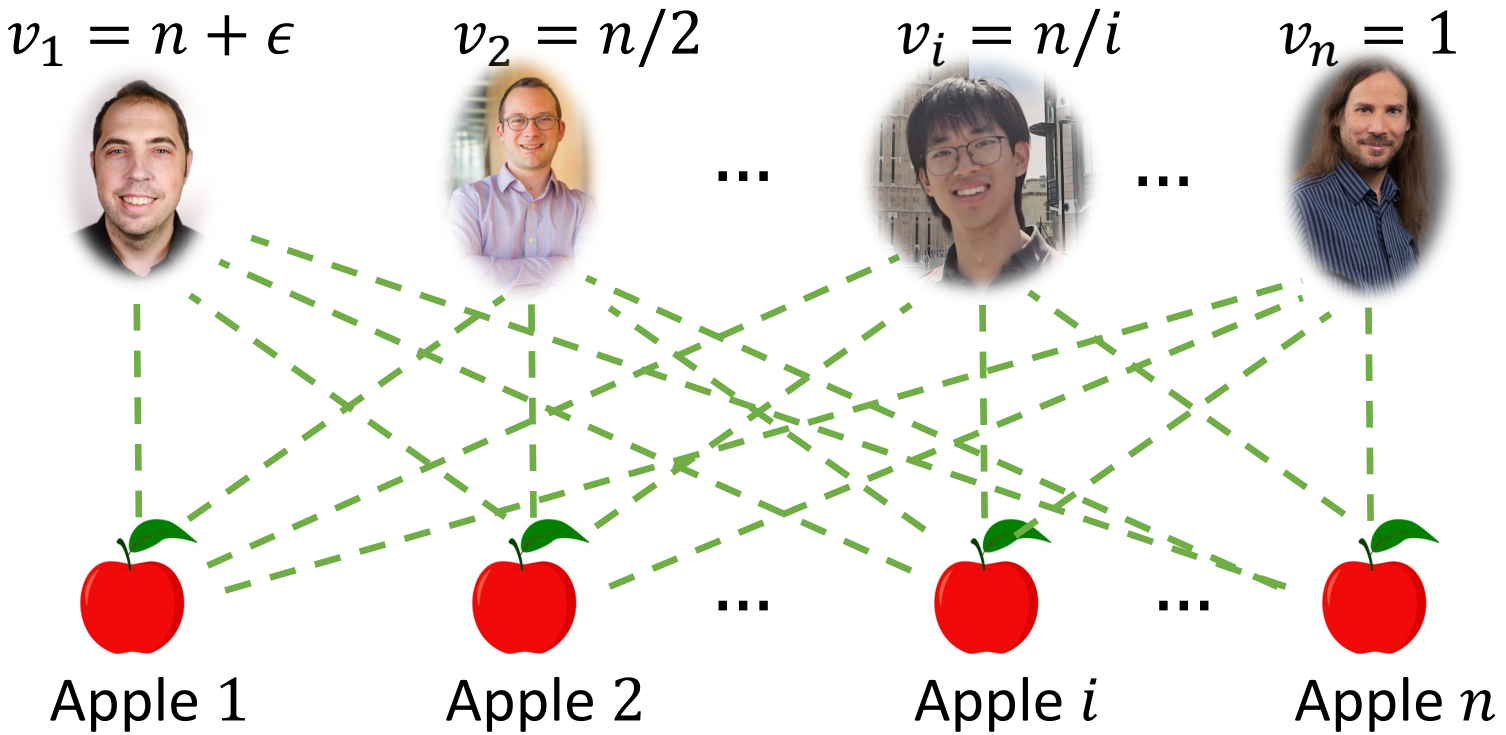

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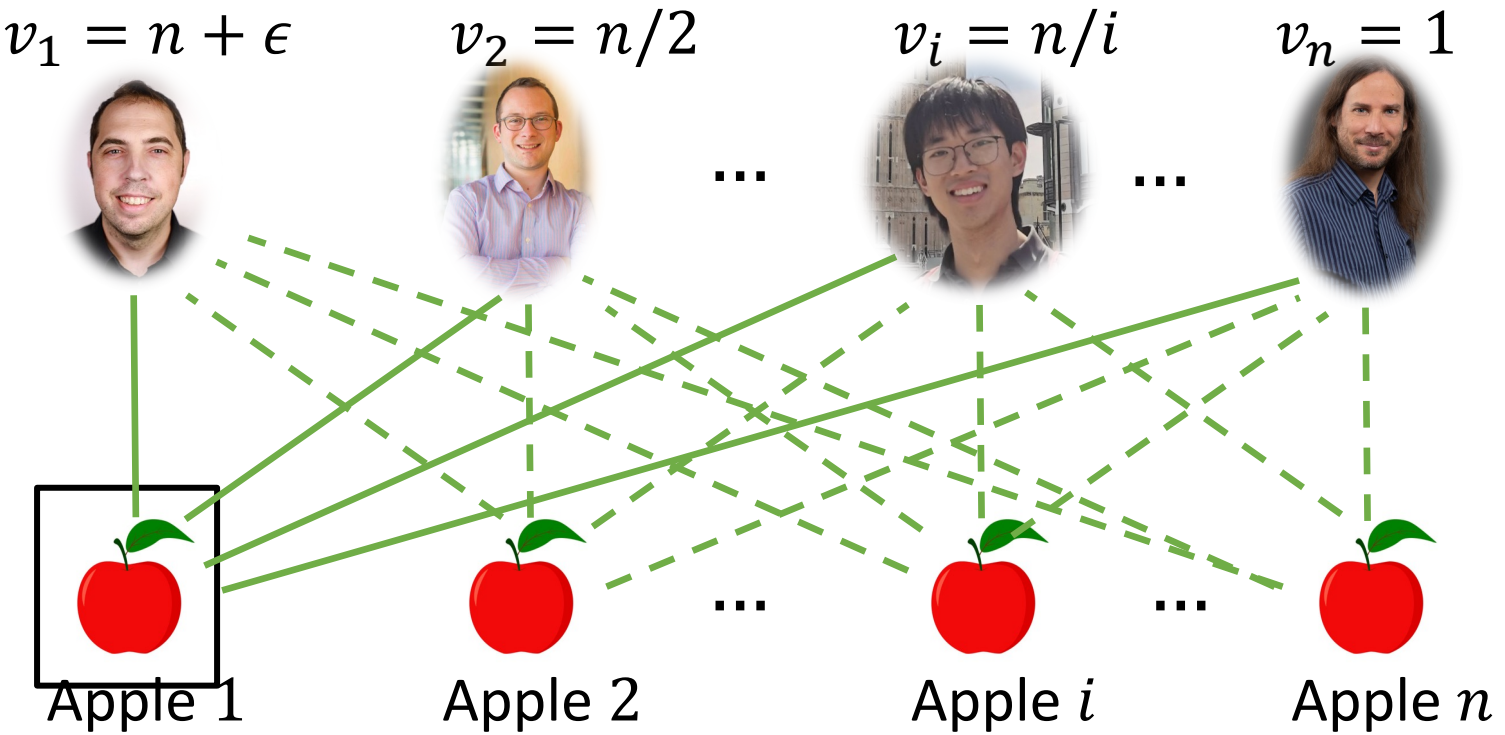

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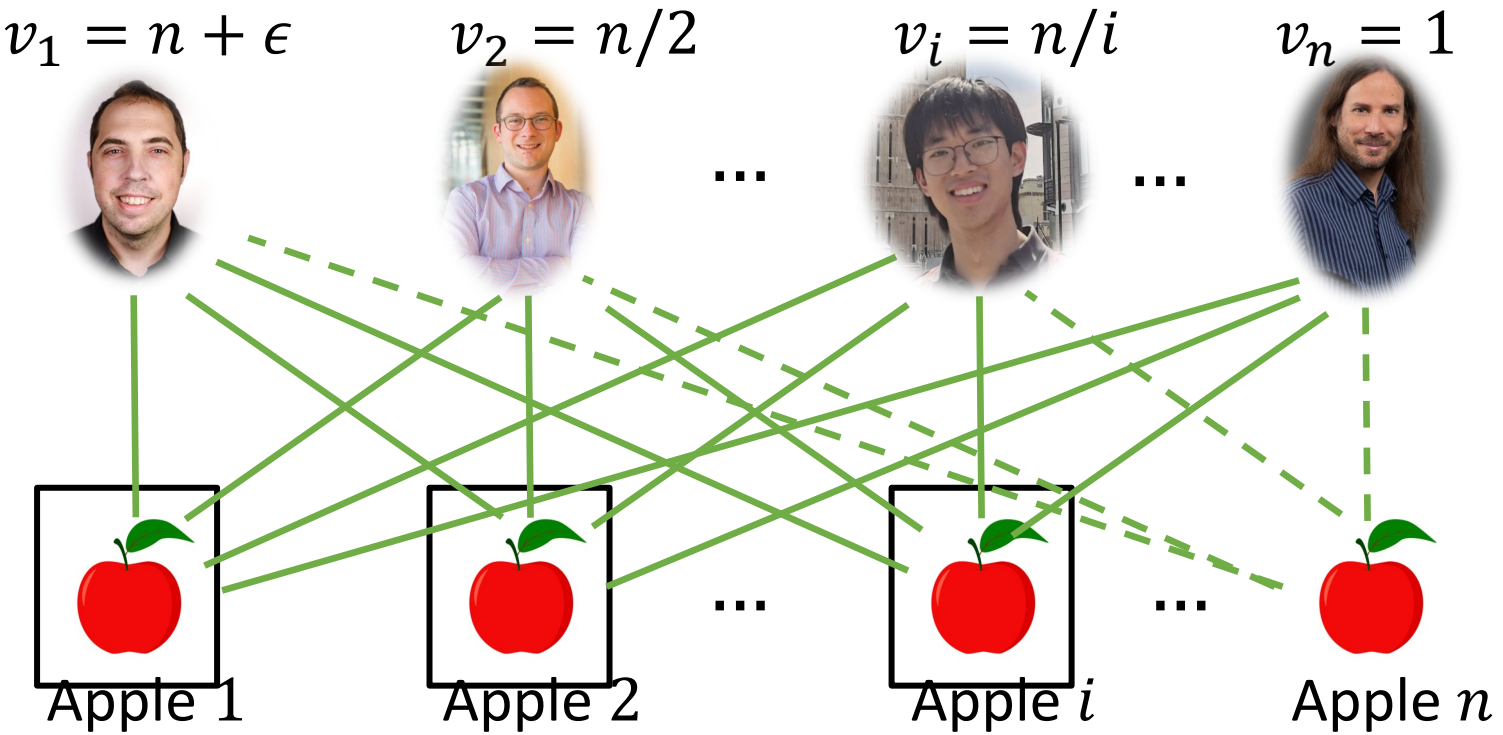

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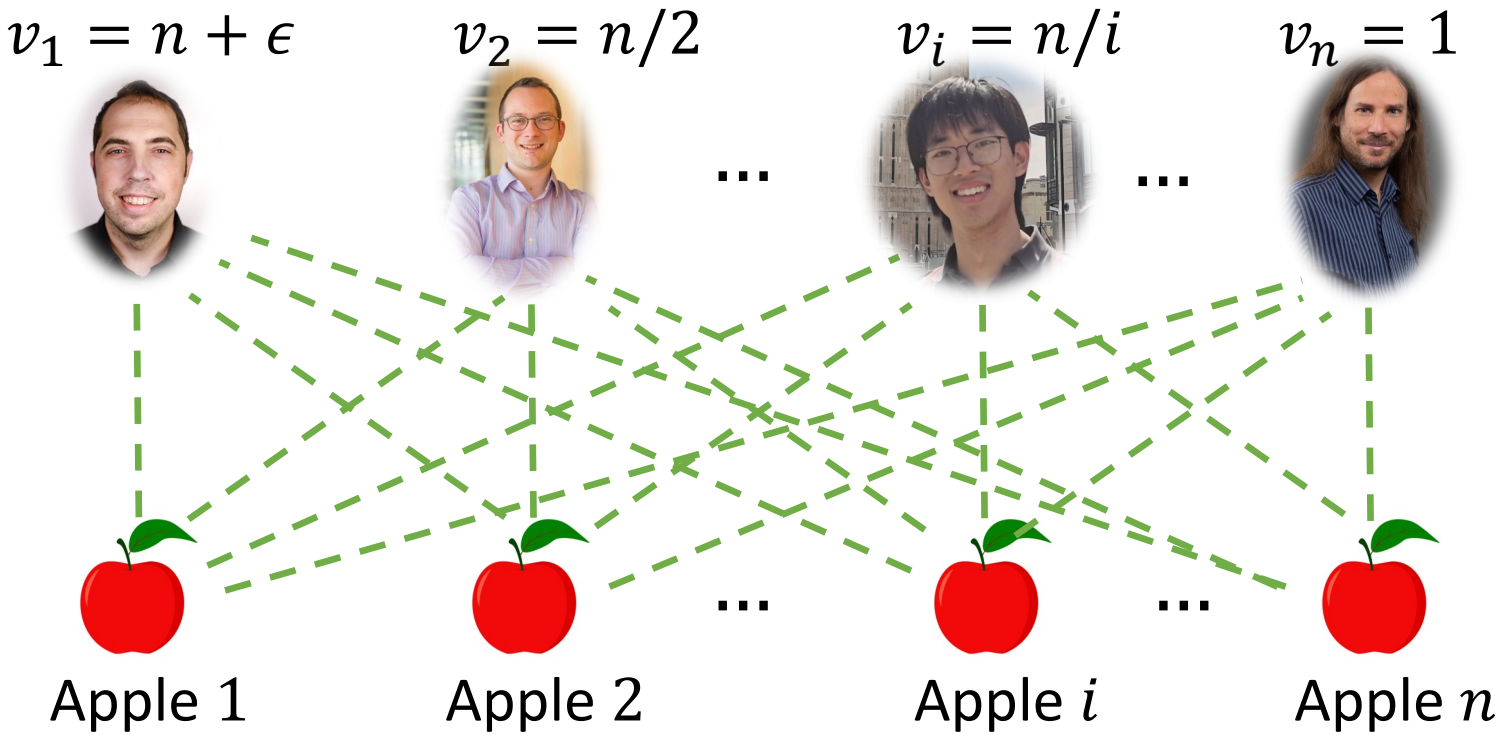

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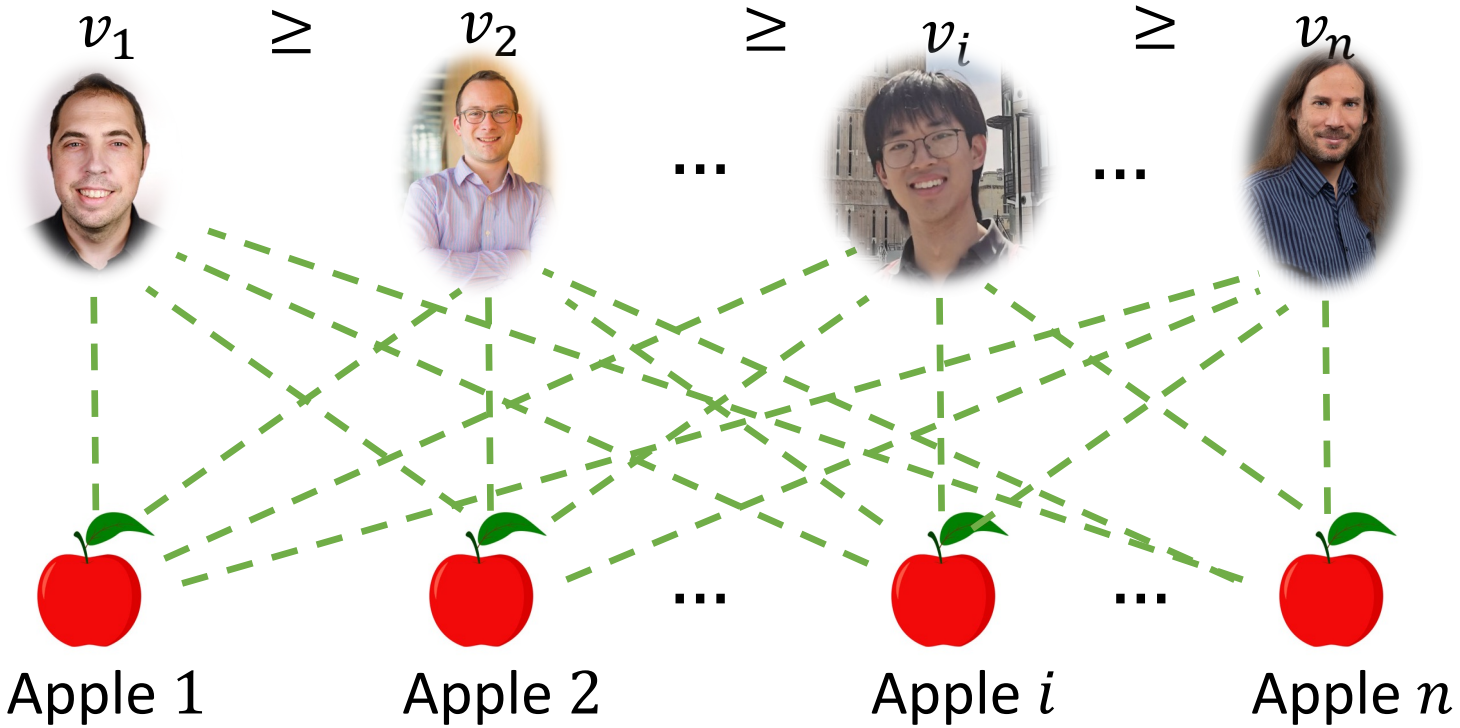
$Eq = n + \epsilon$ $\quad OPT = \epsilon + \sum_i \frac{n}{i} = n \cdot H_n + \epsilon$ $\quad PoA = \Omega(\log n)$

Re2: $O(\log n)$ upper bound (intuition) homo.

Prove welfare guarantee via revenue guarantee



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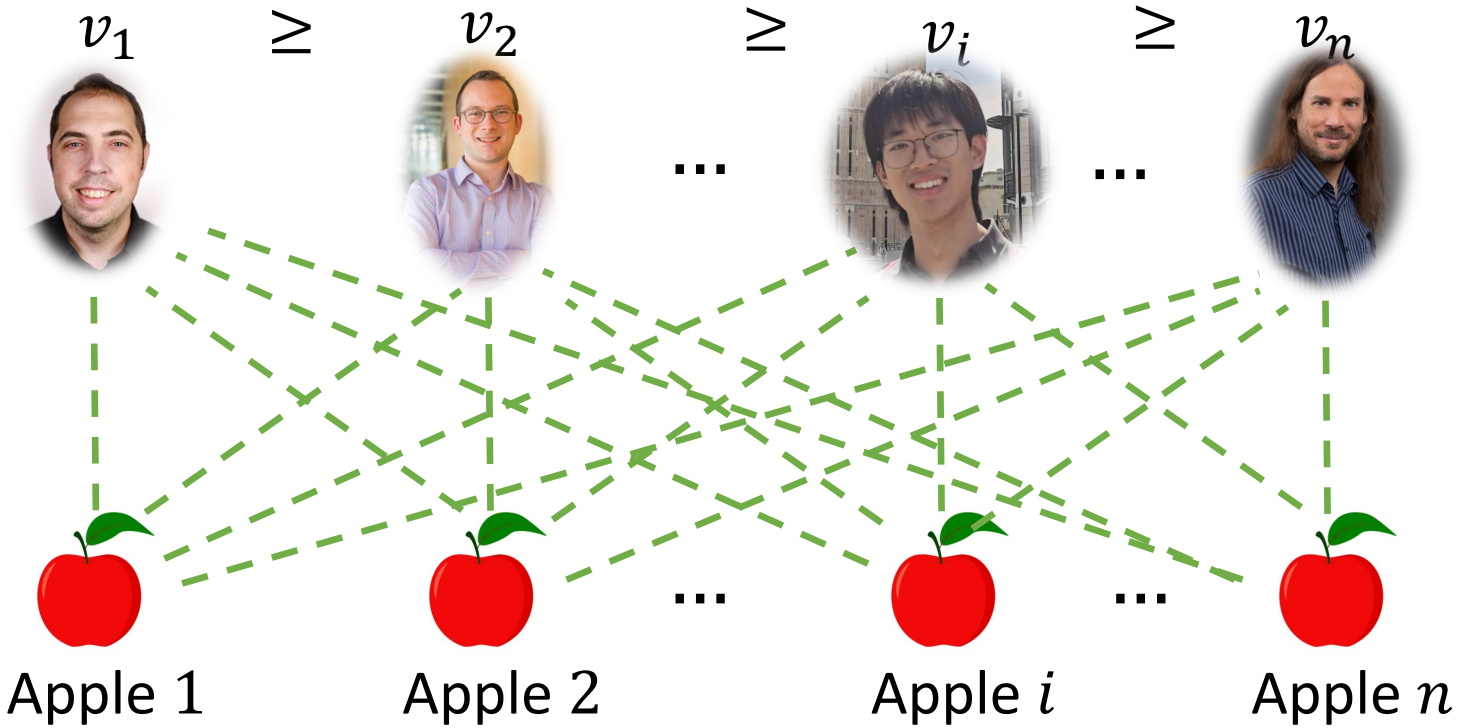

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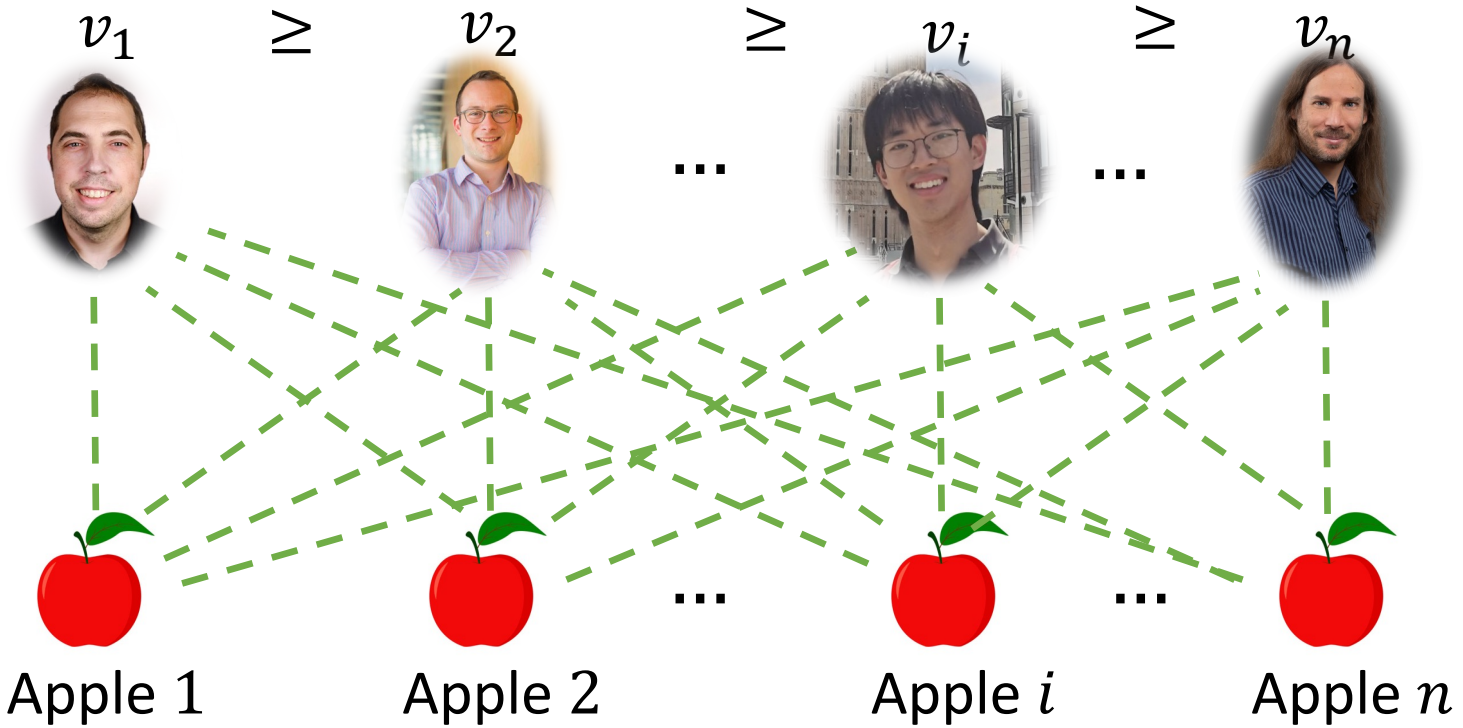

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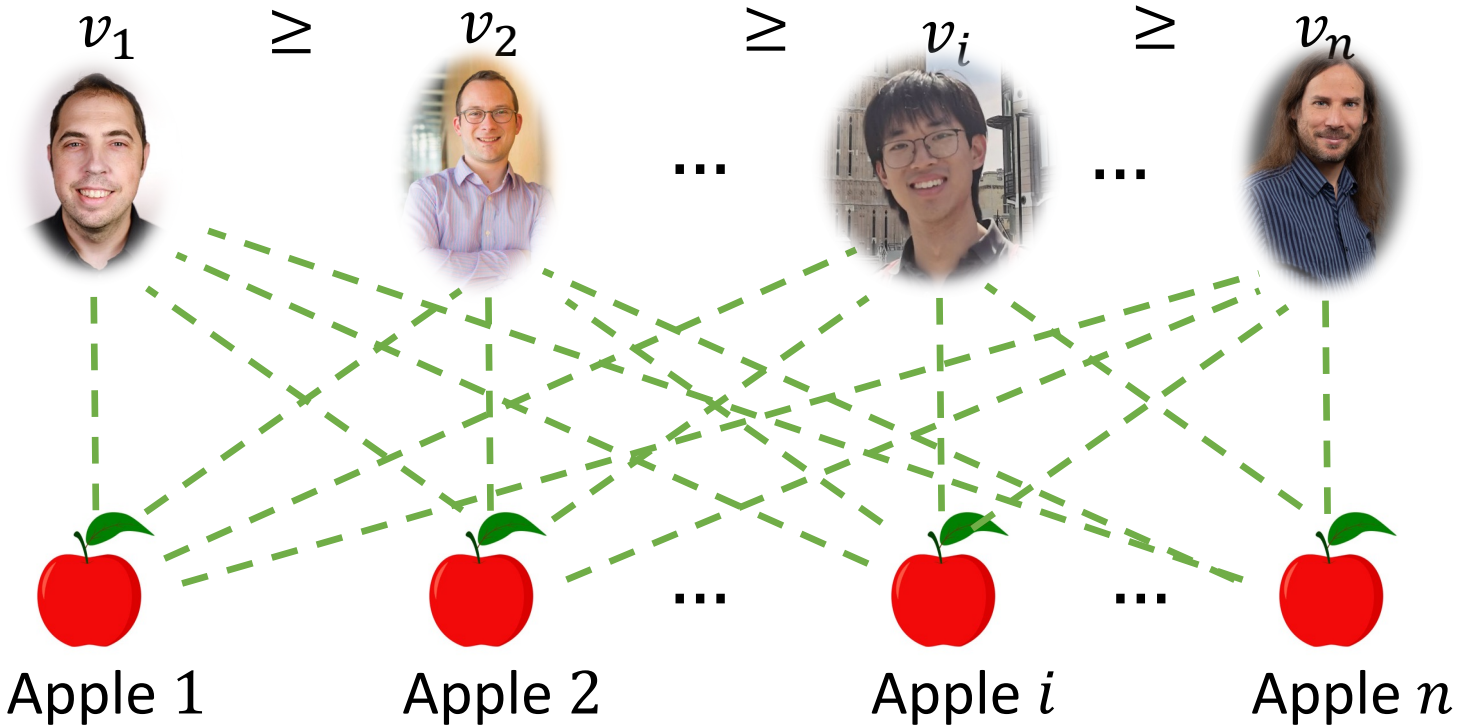

i sellers join: $\text{Rev} = i \cdot v_i$

$$\text{Rev}^* \geq i \cdot v_i \Rightarrow \sum_i \text{Rev}^*/i \geq \sum_i v_i = OPT \Rightarrow \text{Rev}^* \geq OPT/H_n$$

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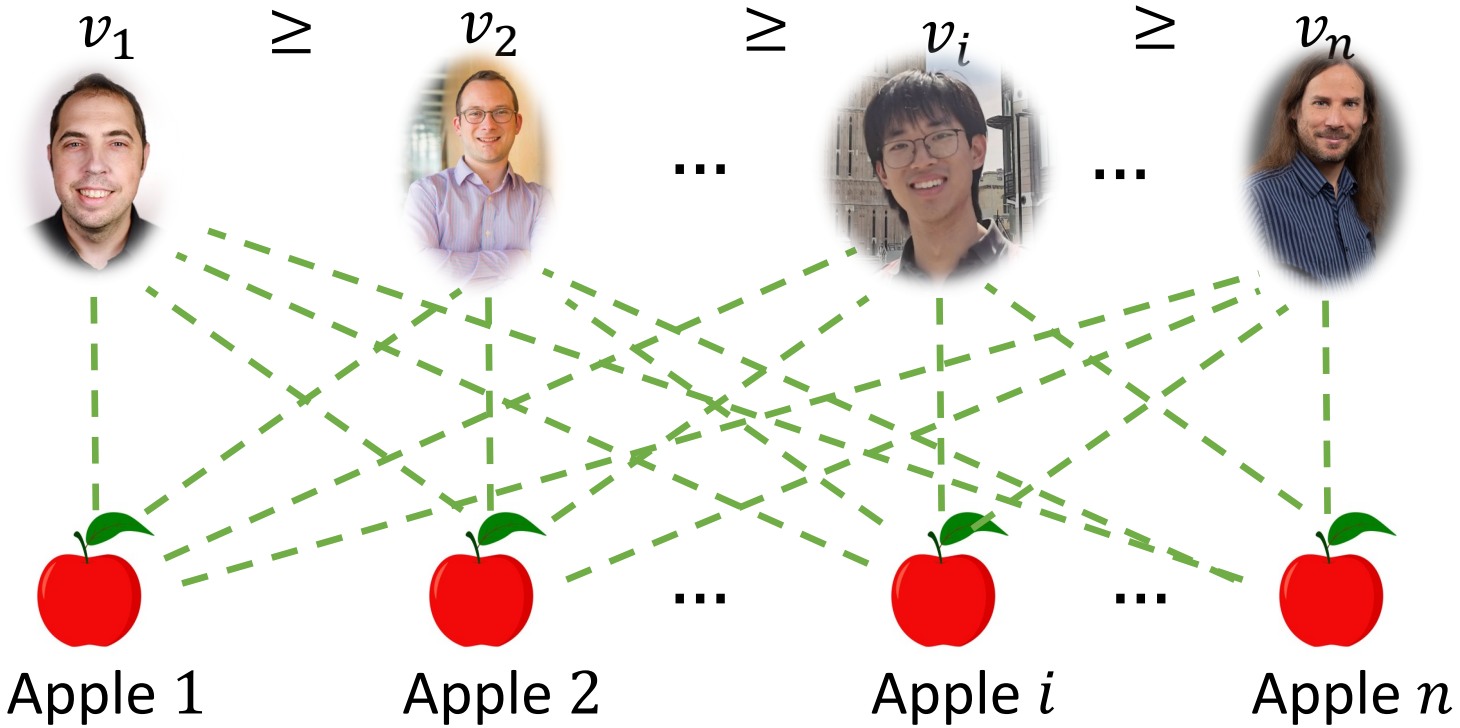

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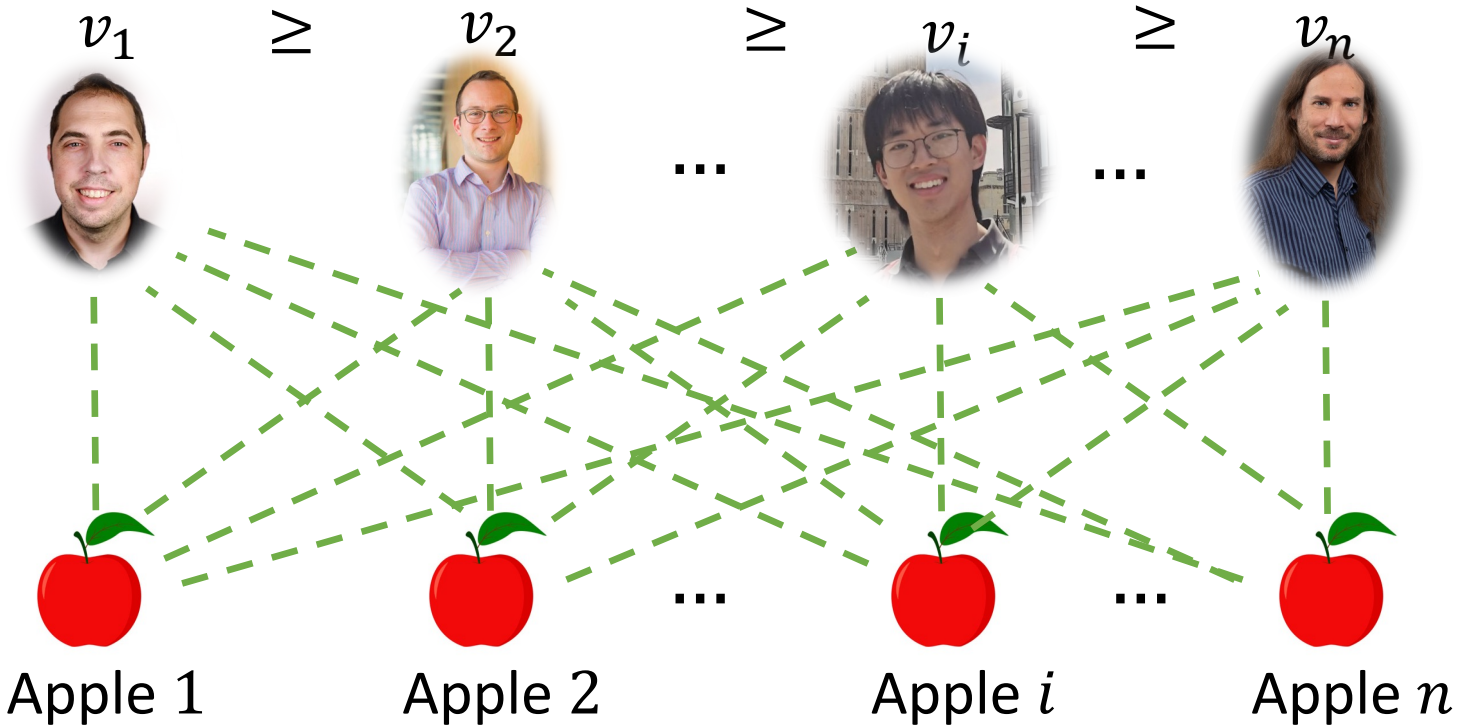
With links



Platform continuously lowers α, pick some number of sellers joining, and lower bound desired α.

Re2: $\Omega\ (n)$ for heterogenous goods

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How regulation helps improve market efficiency?

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Extending to mixed

Given mixed strategies $\boldsymbol{x} = (x_1, \ldots, x_n)$

Define a Bayesian game:

$$\forall i \begin{cases} w.p.\ x_i & i \text{ is connected to all} \\ w.p\ (1 - x_i) & i \text{ uses orignal links} \end{cases}$$

If i joins, i is connected to all, pays fee α

Re3: $\frac{2-\alpha}{1-\alpha}$ for heterogenous valuations

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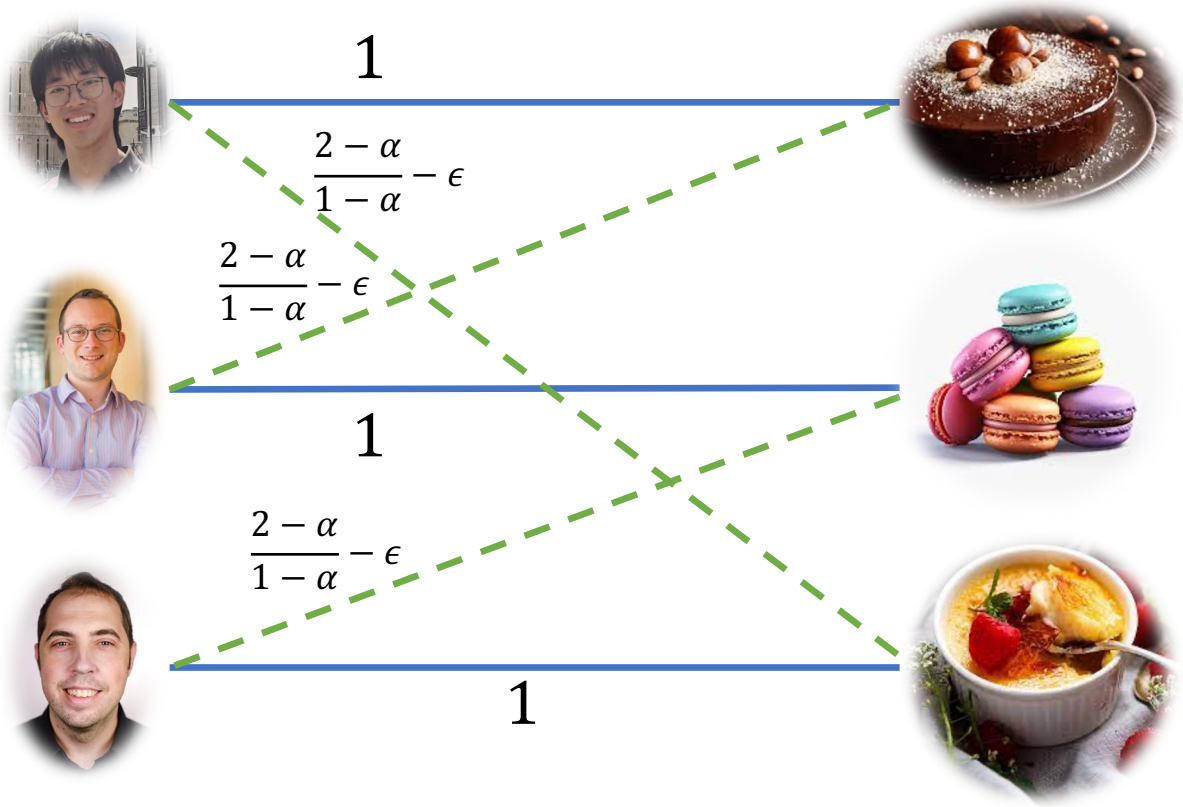
If i joins, i is connected to all, pays fee α

Pure PoA $\left(\frac{2-\alpha}{1-\alpha}\right)$ for the Bayesian game

$\boldsymbol{x}$ is a mixed eq. in the original game ➔
no agents join is a pure eq. in the stochastic game

Re3: $\frac{2-\alpha}{1-\alpha}$ tight instance

α transaction fee



Extensions

- Beyond unit-demand
- Effects of production costs
- Platform matching

Next Steps

- More general valuation
- Competing platforms
- ?

One Liner

Under slight regulation,
platforms can give robust welfare guarantees