School Capital, Funding, and Student Choice

William Phan[1] Ryan Tierney[2] Yu Zhou[3]

[1]North Carolina State University

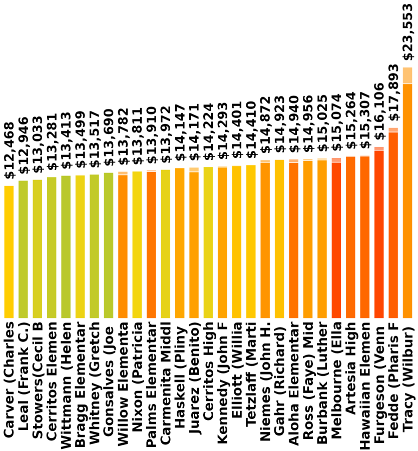
[2]University of Southern Denmark

[3]Nagoya University

From matchings to markets. A tale of Mathematics, Economics and Computer Science. CIRM, 2023.

Funding

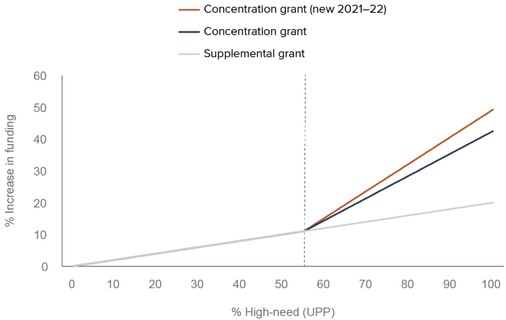
ABC Unified spending per student by school (2020-2021)



Funding

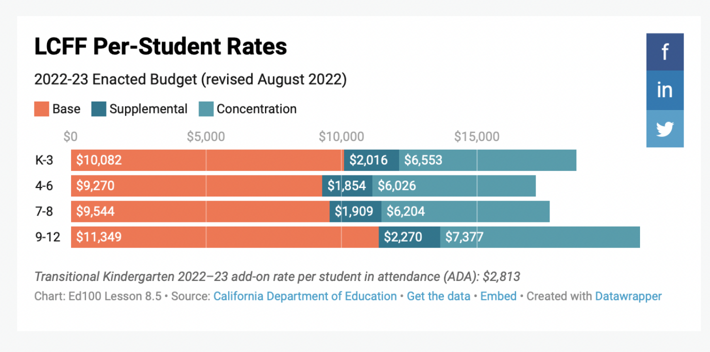
Figure 1

LCFF directs additional funding based on a district's share of high-need students



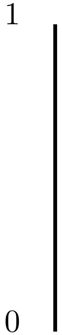
SOURCE: California Department of Education.
NOTES: UPP refers to the "unduplicated pupil percentage" of low-income, English Learner, and foster youth in a district. Figure shows the percent increase in base grant funding for a district with a given share of high-need students.

Funding


LCFF Per-Student Rates
2022-23 Enacted Budget (revised August 2022)
Base
Supplemental
Concentration
$0
$5,000
$10,000
$15,000
K-3
$10,082
$2,016
$6,553
4-6
$9,270
$1,854
$6,026
7-8
$9,544
$1,909
$6,204
9-12
$11,349
$2,270
$7,377
Transitional Kindergarten 2022–23 add-on rate per student in attendance (ADA): $2,813
Chart: Ed100 Lesson 8.5 • Source: California Department of Education • Get the data • Embed • Created with Datawrapper
f
in

Contribution

1. A model of *resources* and school choice.
2. Propose *enrollment-based funding equilibria*.
3. Applications
   3.1 Progressive funding in schools
   3.2 Peer effects
   3.3 Vouchers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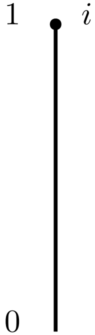
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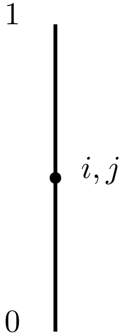
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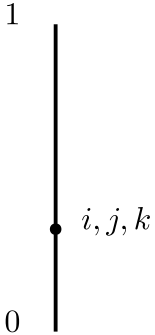
1
i, j, k
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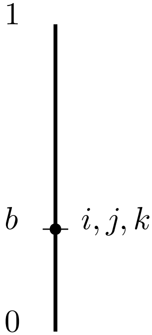
1
b
i, j, k
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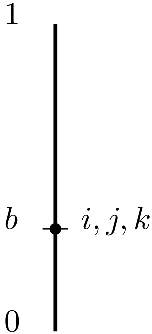
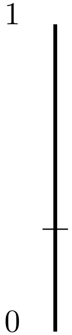


$\frac{1}{b} = capacity$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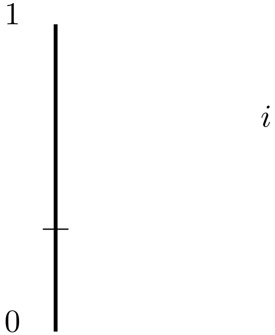
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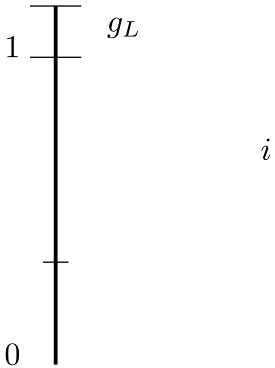

g_L
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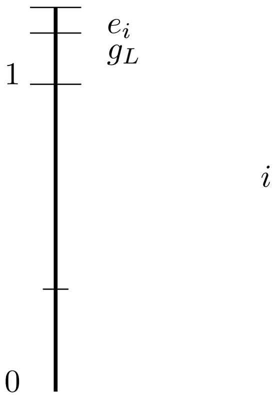

e_i
g_L
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i
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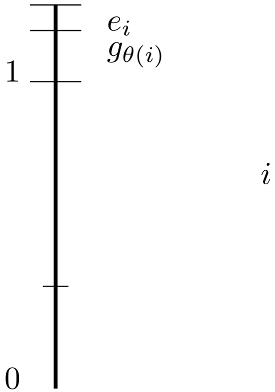
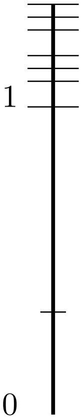
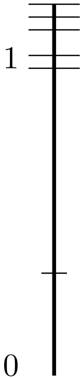


e_i
$g_{\theta(i)}$
1
i
0



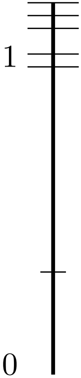
$$F_s(\sigma[s]) = \sum_{i \in \sigma[s]} g_{\theta(i)} + e_i$$

$$\sigma[s] = i, j, k$$



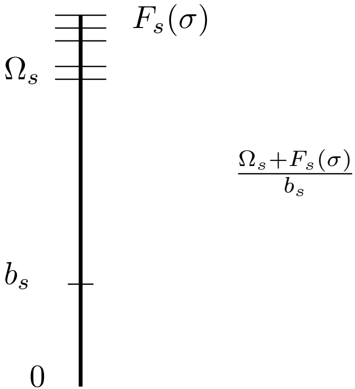
$$F_s(\sigma[s]) = \sum_{i \in \sigma[s]} g_{\theta(i)} + e_i$$

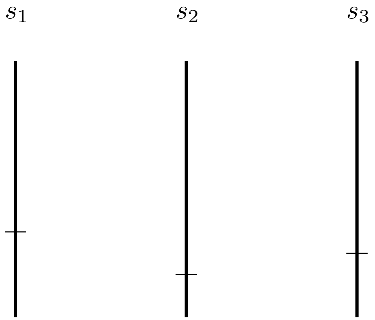
$$\sigma[s] = i, j, \ell$$

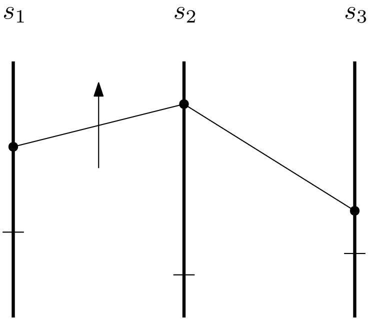


$$F_s(\sigma[s]) = \sum_{i\in\sigma[s]} g_{\theta(i)} + e_i$$

$$\frac{1+F_s(\sigma)}{b_s}$$


$F_s(\sigma)$
Ω_s
$\frac{\Omega_s+F_s(\sigma)}{b_s}$
b_s
0


s_1
s_2
s_3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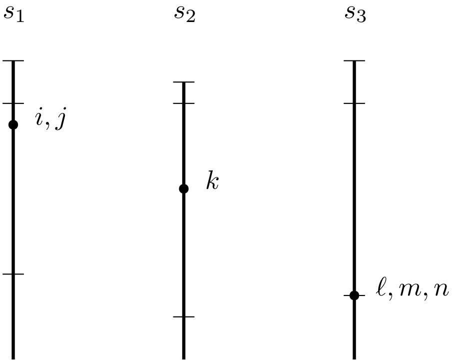
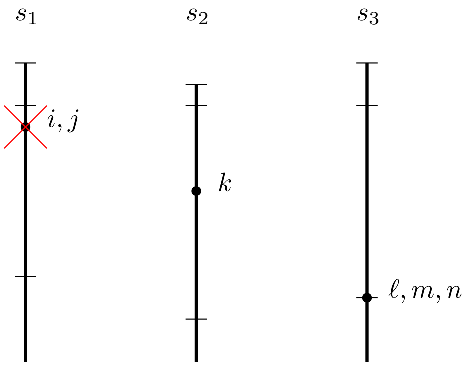


s_1
s_2
s_3
i, j
k
ℓ, m, n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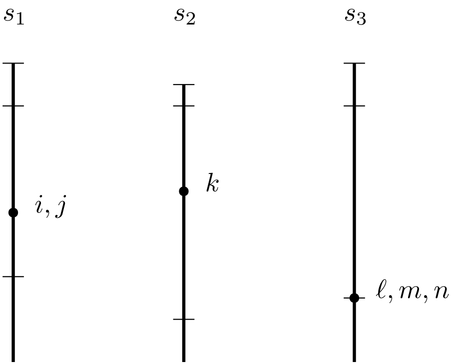
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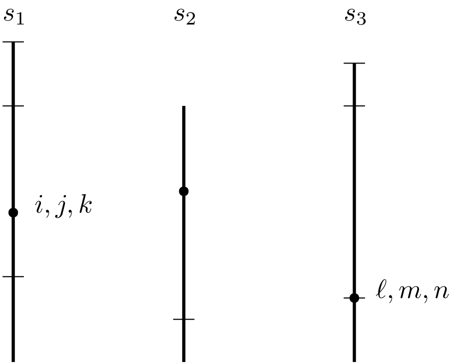


s_1
s_2
s_3
i, j, k
ℓ, m, n

A **student's externality** has 3 parts:

1. negative crowding effect
2. positive funding
3. negative/positive personal peer effect

Model

L, H	low and high income students
S	schools
$\mathbb{R}_+ \times S$	consumption space of students
R_i	preference relation of student i
Ω_s	capital stock of school s
b_s	minimum resource level of school s
g_L	low-income grant
g_H	high-income grant
e_i	peer effect of i
$\succ_s$	priority order of school s over students

$(L, H, S, R, \Omega, b, g, e, \succ)$	a **school choice with funding problem**

ρ_s	resource level of school s
$\sigma(i)$	match of student i
(ρ, σ)	allocation

Model

Let

1. $0 \leq g_{\theta(i)} + e_i < b_s$
2. $F_s(\sigma) = \Omega_s + \sum_{i \in \sigma[s]} g_\theta(i) + e_i$

An **allocation** $(\boldsymbol{\rho}, \sigma)$ is such that for each school s,

1. (Distribution Feasibility) $\rho_s \cdot |\sigma[s]| \leq F_s(\sigma)$.
2. (Minimum Bound) $\rho_s \geq b_s$.

Axiom

Fairness

No Envy at Interior + No Justified Envy at Boundary

Axiom

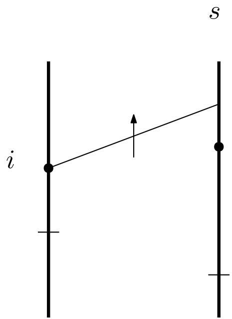
Fairness
If student i prefers school s to their match at the current level, then s is

1. full, and
2. all students at s have higher priority.

Axiom

Fairness
If student i prefers school s to their match at the current level, then s is

1. is at minimum b_s, and
2. all students at s have higher priority.


s
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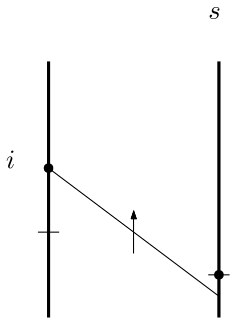
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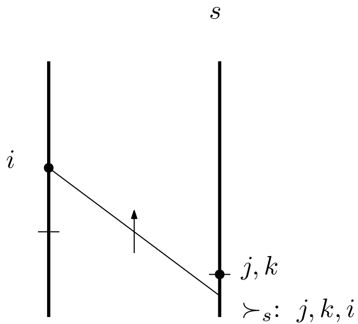

s
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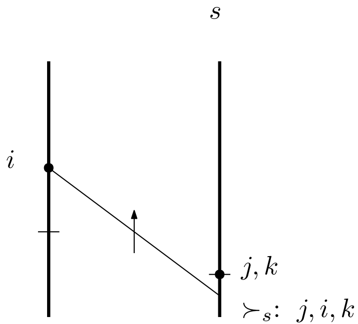
j, k
≻s: j, k, i



s
i
j, k
$\succ_s$: j, i, k

Axiom

Fairness:

$$(\boldsymbol{\rho}, s) \; P_i \; (\boldsymbol{\rho}, \sigma(i)) \quad \Rightarrow \quad \begin{array}{l} 1.\ \rho_s = b_s, \text{ and} \\ 2.\ \text{for each } j \in \sigma[s], \\ \quad\ j \succ_s i \end{array}$$

Fair Lattice

Theorem 1 Consider preference profiles on the NCBI domain. Let (ρ, σ) and (γ, τ) be two *fair* allocations. Then, there is matching μ such that allocation $(\boldsymbol{\rho} \vee \boldsymbol{\gamma}, \mu)$ is *fair*, and for each $i \in L \cup H$,

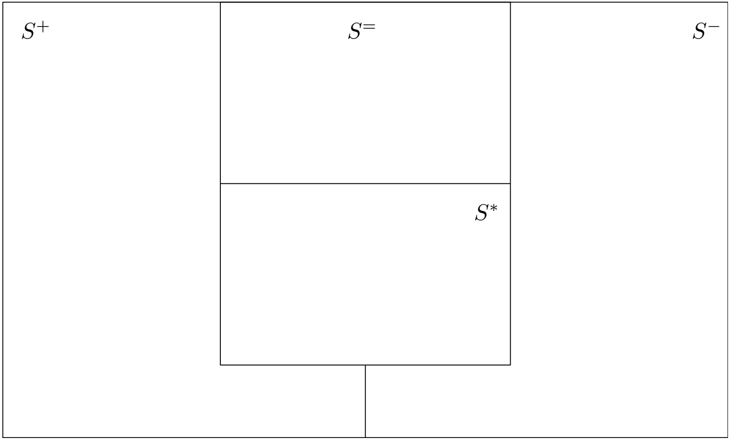
$$(\boldsymbol{\rho} \vee \boldsymbol{\gamma}, \mu(i)) \; R_i \; \max_{R_i}\{(\boldsymbol{\rho}, \sigma), (\boldsymbol{\gamma}, \tau)\}.$$

Fair Lattice

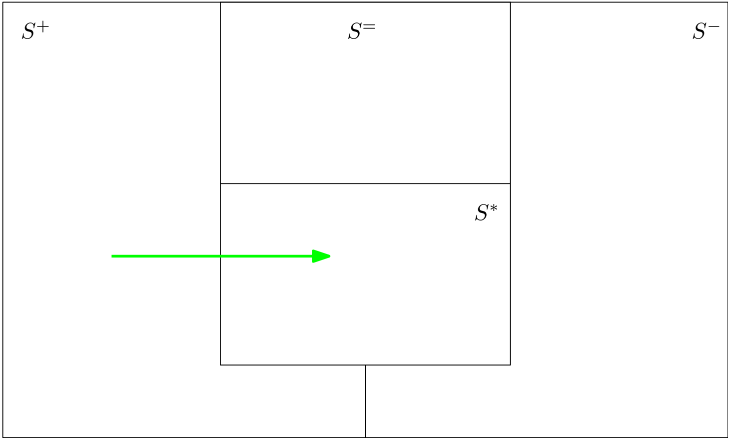
Sketch Proof:

1. Pick two fair allocations.
2. Define Transfer graph.
3. Implement weakly welfare-increasing cycles.
4. (Key) Make sure you maintain feasibility.

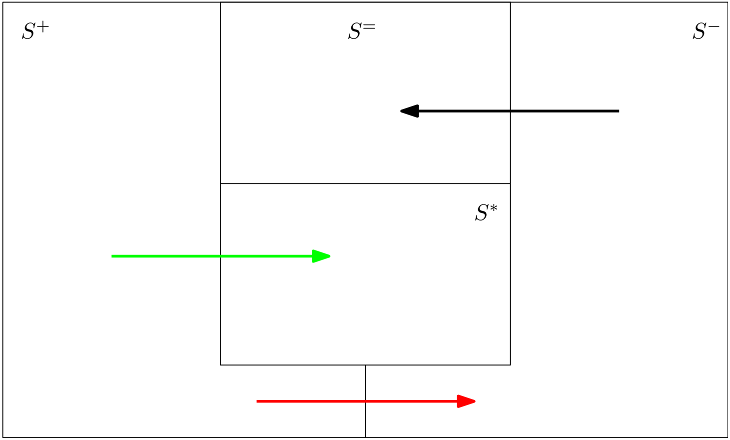
Fair Lattice



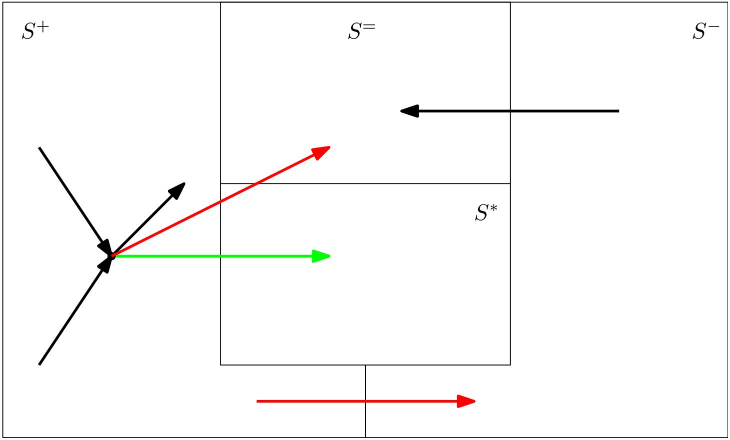
Fair Lattice



Fair Lattice



Fair Lattice



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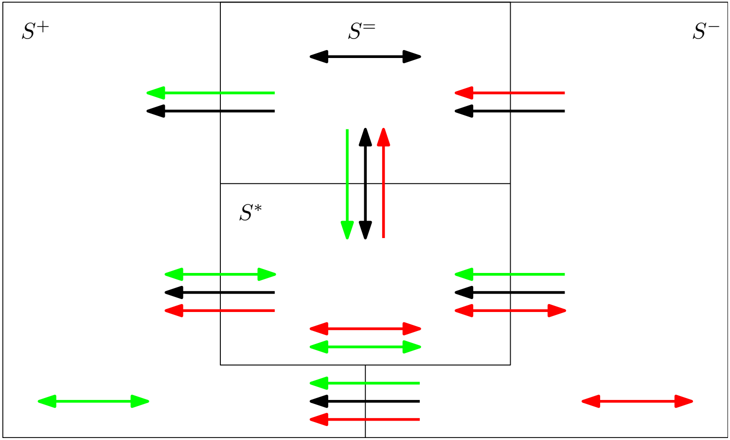

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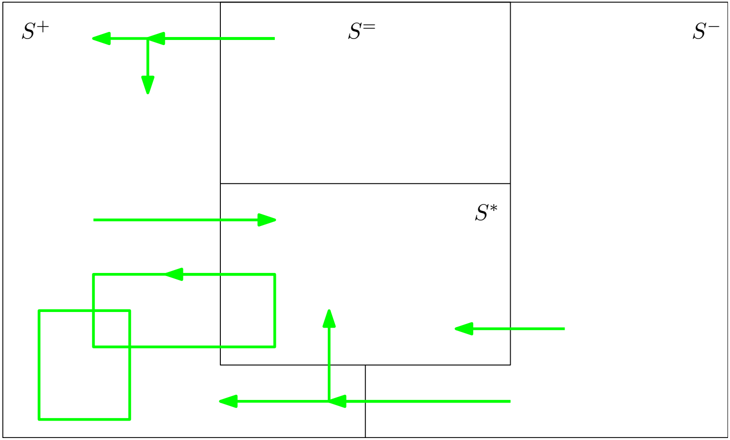

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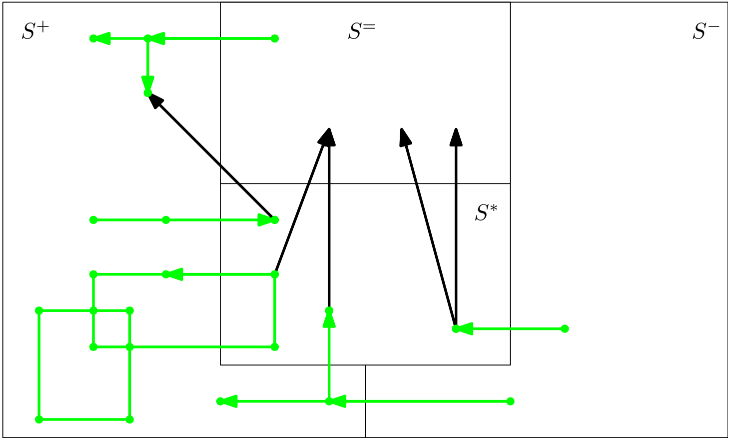

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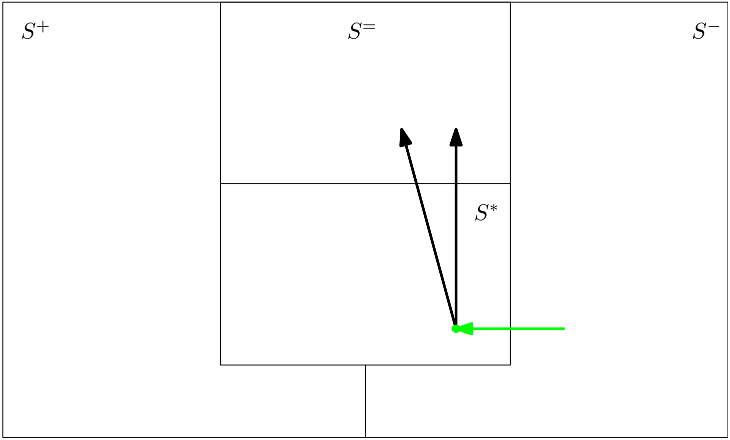

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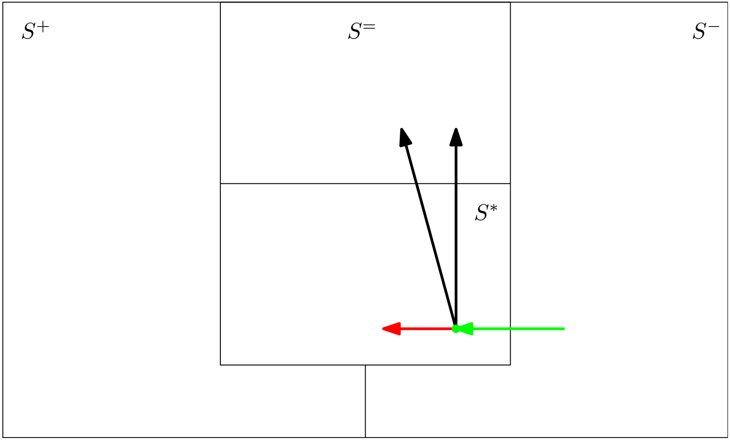

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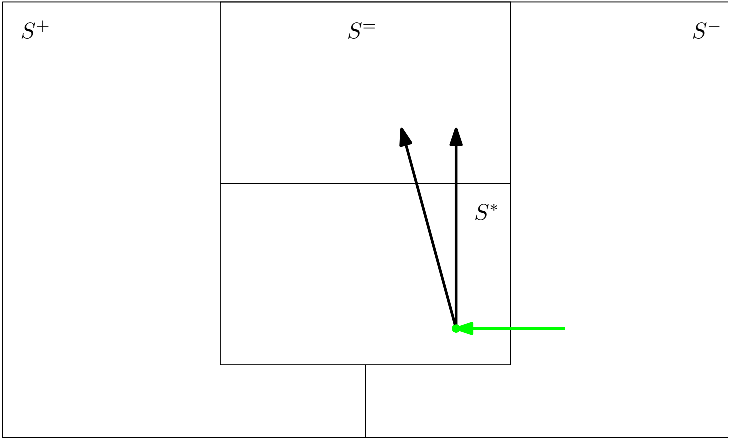

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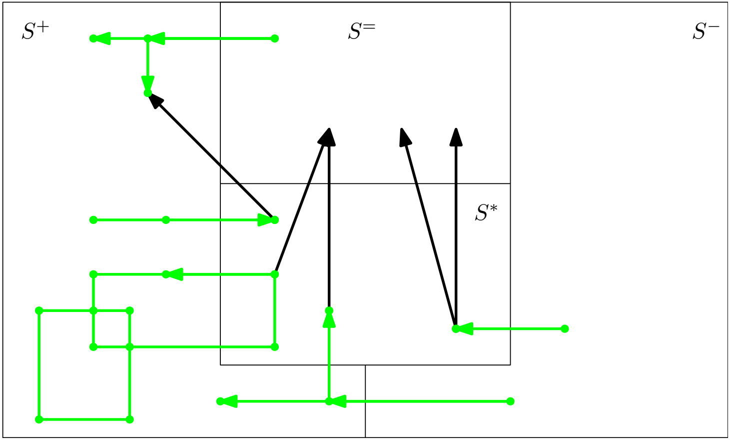

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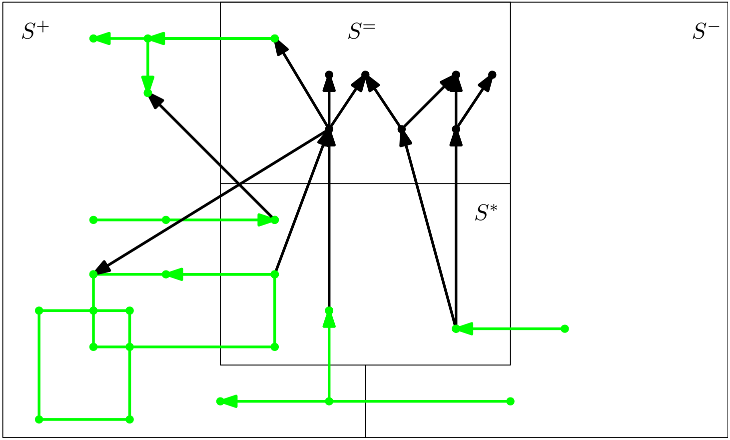

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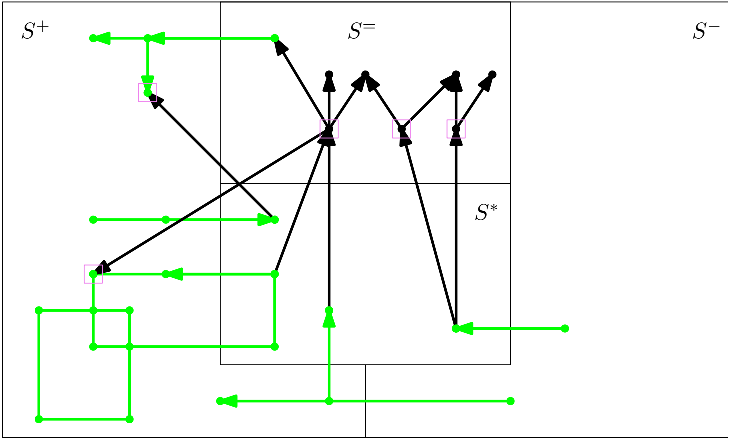
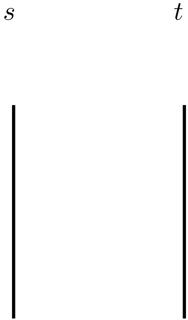
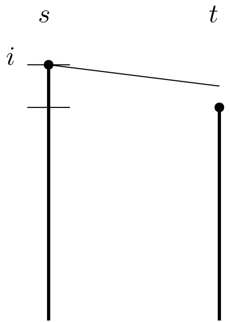


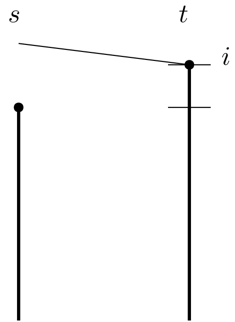
No Rural Hospital Theorem



No Rural Hospital Theorem



No Rural Hospital Theorem



Enrollment-Based Funding Equilibrium (EBFE)

An EBFE is an allocation $(\boldsymbol{\rho}, \sigma)$ such that:

1. (Fairness) $(\boldsymbol{\rho}, \sigma)$ is *fair*.
2. (Exhaustive Given $\boldsymbol{\rho}$) For each school s with $\sigma[s] \neq \emptyset$,

$$\left\lfloor \frac{F_s(\sigma)}{\rho_s} \right\rfloor = |\sigma[s]|.$$

3. (Inferior Empty Schools) For each school s with $\sigma[s] = \emptyset$, $\rho_s = \Omega_s$, and for each $i \in N$,

$$(\boldsymbol{\rho}, \sigma(i)) \; P_i \; (\boldsymbol{\rho}, s).$$

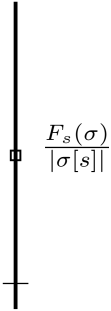
EBFE Is a Walrasian Concept

An EBFE is an allocation $(\boldsymbol{\rho}, \sigma)$ such that:

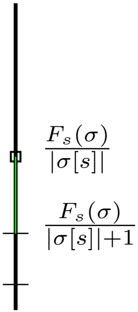
1. (Fairness) Consumer maximization.
2. (Exhaustive Given $\boldsymbol{\rho}$) Market clearing with one seat error.
3. (Inferior Empty Schools) Set price of unassigned good to zero.

Price Equilibrium with Rigidities
Drèze [1975], Talman and Yang [2008], Andersson and Svensson [2014], Herings [2018].

Exhaustiveness Discrepancy



Exhaustiveness Discrepancy



Existence

A preference is **linear** if $(r, s)\ R_i\ (g, t)$ implies that there is valuation function $v_i : S \to \mathbb{R}_+$, such that

$$(r, s)\ R_i\ (g, t) \iff rv_i(s) \geq gv_i(t).$$

Existence

Theorem 2 On the NCBI linear domain, EBFE exist (and are the maximal *fair* allocations).

Extension: Vouchers

L, H	low and high income students
S	schools
$\mathbb{R}_+ \times S \times \mathbb{R}_+$	consumption space of students
R_i	preference relation of student i
Ω_s	capital stock of school s
b_s	minimum resource level of school s
v_i	voucher of i
g_L	low-income grant
g_H	high-income grant
e_i	peer effect of i
$\succ_s$	priority order of school s over students
$(L, H, S, R, \Omega, b, v, g, e, \succ)$	**school choice with funding problem**

Literature

School Choice
Abdulkadiroğlu and Sönmez [2003], Phan et al. [2023].

Matching with Externalities
Sasaki and Toda [1996], Hafalir [2008], Bando [2012], Bando [2014], Dutta and Massó [1997], Echenique and Yenmez [2007], Roth [1984], Aldershof and Carducci [1996], Roth and Peranson [1999], Klaus and Klijn [2005], Kojima et al. [2013], Ashlagi et al. [2014], Dur and Wiseman [2019], Dur et al. [2022], Pycia [2012], Rostek and Yoder [2020], Pycia and Yenmez [2021].

Literature

Price Equilibrium with Rigidities
Drèze [1975], Talman and Yang [2008], Andersson and Svensson [2014], Herings [2018].

Price As Cutoff Vector
Balinski and Sönmez [1999], Sönmez and Ünver [2010], Azevedo and Leshno [2016], Dur and Morrill [2018].

Crowding
Tierney [2019].

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