General information for the conference

Welcome!

Wifi: you should have received an email

Lunch: 12:30pm everyday

Diner: 7:30pm

 \Rightarrow Special diner on Thursday: Bouillabaisse

Wed afternoon: free for all

⇒ Feel free to create a group for hiking Hope you will have a nice and productive time!



Call for applications Postdoctoral Fellowships 2024

Deadline for applications: January 15, 2024 at noon

Hi! PARIS is pleased to announce and present our Call for Postdoctoral Fellowships.

Hi! PARIS is the new interdisciplinary Center on Data Analytics and Artificial Intelligence for Science, Business and Society created by Institut Polytechnique de Paris (IP Paris) and HEC Paris and recently joined by Inria (Centre Inria de Saclay). One of the central aims of Hi! PARIS is to conduct breakthrough and multidisciplinary research on AI and Data Science.

Market Design for Distributional Objectives in (Re)assignment: An Application to Improve the Distribution of Teachers in Schools

Julien Combe CREST & École Polytechnique Umut DurOlivier TercieuxNC State UCNRS & PSE

Camille Terrier QMUL Utku Ünver Boston College

under revision

From matching to markets CIRM - Marseille

Dec 11-15, 2023





Centralized (Re)assignment

Centralized (re)assignment involves

first time assignment of new workers to jobs together with reassignment of senior workers who would like to move to a different job.

Examples:

Government Sector: Police officers (e.g. Chicago), doctors (many countries), administrators (e.g. India), teachers (many countries) Private Sector: Job rotations (many large corporations)

Common features:

- 1. One or few large employers are in charge of jobs.
- 2. Workers have preferences over jobs.
- 3. Employers have distributional objectives
- Senior workers can stay at their job or move to a better one; new workers need a first-time job.

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Distributional objectives exist, yet can sometimes be in conflict with agents preferences.

Examples:

Senior police officers shy away from urban areas, CPD needs more officers in urban areas due to disproportionate crime rates (Sidibe et al., 2021).

Indian civil servants often get assigned close to their home states, while the government needs them to be distributed around for national integration (Thakur, 2020).

Main application in this paper:

Disadvantaged regions have relatively more inexperienced teachers; to decrease the education achievement gap in the country, more experienced teachers are needed in these regions.

- Empirical evidence points that experience level of teachers positively affect education outcomes. (Chetty, Friedman, Rockoff, 2014 – in the U.S.)
 - (Allen, Mian, Sims, 2016 in the U.K.)

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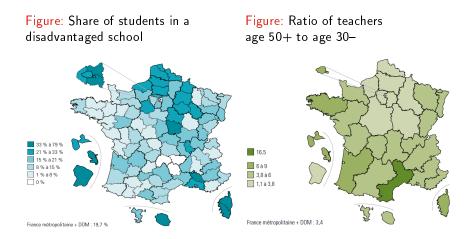
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Example: Teacher Distribution in France



Contribution

Contribution 1: We propose a new mechanism which incentivizes truthful reports from teachers and improves both schools and teachers with respect to a status-quo matching. A school improvement is measured by a (Lorenz) shift of the types' distribution of its assigned teachers following a priority ordering over types (e.g. ranking over exp. levels)

Contribution 2: In a large market setting, we show how a global objective of decreasing inequalities across schools can be achieved by designing priorities for schools and using our proposed mechanism to shift their types' distribution

Contribution 3: Using French data, we conduct empirical simulations: our mechanism achieves a decrease in inequalities while other benchmarks do not, notably those without distrib. objectives

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2 Model

- 3 School priorities and Lorenz Dominance
- 4 SI-CC mechanism and its properties
- 5 Priority design for inequality reduction
- 6 Empirical analysis (short version)

7 Conclusion

The Model

- T: set of n teachers
- N: set of new teachers
- $T \setminus N$: set of tenured teachers

Types: t has an experience level $\theta(t) \in \Omega \subset \mathbb{R}$ where Ω is finite

- S: set of m schools, each school s with quota q_s .
- ω : status-quo matching is the initial allocation

 $\omega_t \in S \cup \{\emptyset\}$: the initial school of teacher t $\omega_t = \emptyset \iff t$ is a new teacher $\omega_s \subseteq T$: the initial employees of school s

 P_t : strict preference relation of teacher t over $S \cup \{\emptyset\}$

Inequality reduction: priorities v.s. global objective

Main objective: reduce inequalities across schools

\Rightarrow Global objective that is complex

Priority design: how to achieve the global objective of inequality reduction using priorities for each school?

⇒ Intuition: "shift up" the distrib. of schools with low exp. and "shift down" the one of schools with high exp.

Simpler: improve each school's distribution of experience according to some fixed priority ordering over experience levels

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School Priorities and Lorenz Dominance

\triangleright_s : priority over teacher types for school s (a linear order)

Comparisons over sets of teachers are based on Lorenz comparison, i.e., using first-order stochastic dominance of type distributions assigned: For any two sets of teachers \bar{T}, \hat{T}

$$\forall \theta \quad \sum_{\theta' \succeq_s \theta} \# \text{ type-} \theta' \text{teachers in } \bar{T} \ \geq \ \sum_{\theta' \succeq_s \theta} \# \text{ type-} \theta' \text{teachers in } \hat{T}$$

$$\iff \qquad \bar{T} \succeq_s \hat{T}$$

 \Rightarrow Will be justified by priority design for inequality reduction (next section).

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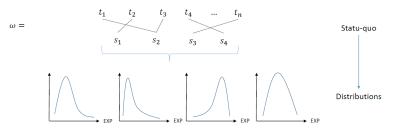
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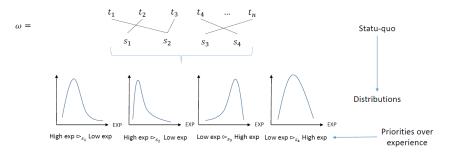


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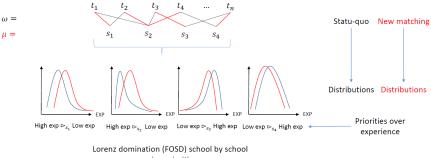


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School priorities and Lorenz Dominance

School Priorities and Lorenz Dominance



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using priorities

A mechanism φ maps teacher preferences to matchings

arphi is individually rational (IR) if for every profile P,

for every teacher $t - \varphi_t(P) R_t \omega_t$.

 φ is school improving (SI) if for every profile P and school s: $\varphi_s(P) \succeq_s \omega_s$ (Lorenz domination)

 \Rightarrow With the right priority \triangleright : decreases ineq. accross schools

 φ is strategy-proof (SP) if for every profile P, teacher t, and manipulation \hat{P}_t : $\varphi_t(P_t, P_{-t}) R_t \varphi_t(\hat{P}_t, P_{-t})$.

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SI Cycles and Chains (SI-CC) Mechanism

School pointing rule: Each school points to its employees in reverse order of its priority ranking (using a fixed tie breaker).

Teacher pointing rule: Each teacher *t* points to her top-choice school *s* such that

either

Replacing the teacher that school s is pointing with teacher t weakly improves school s w.r.t. the status quo

or

school *s* has a vacant seat.

Chain construction & selection rule: Only chains

beginning with a new teacher and

ending at a school with a vacant position

are selected (when no SI cycle exists), so that schools do not lose employees without replacing them.

Formal Definition 🚺 Skip the SI-CC Example

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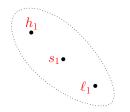
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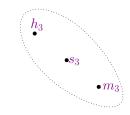
4 schools: s_1 , s_2 , s_3 , s_4 $q_{s_1} = q_{s_3} = 2$ and $q_{s_2} = q_{s_4} = 1$ 3 teacher types: high (h), medium (m), low (ℓ) experiences 6 teachers: 3 high , 2 medium , 1 low type status-quo matching: h_1 and ℓ_1 at s_1 m_2 at s_2 h_2 and m_2 at s_2

 h_3 and m_3 at s_3 h_N new teacher no teacher at s_4

h_1	ℓ_1	m_2	h_3	m_3	h_N
		s_4	s_1	s_2	s_2
s_2		s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

	s_1	s_2	s_3	s_4
	h	h	ℓ	l
	m	m	m	m
	ℓ	l	h	h
status quo:	h_1, ℓ_1	m_2	m_3, h_3	Ø
current:	h_1,ℓ_1	m_2	m_3, h_3	Ø









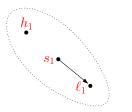


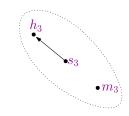
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h_1	ℓ_1	m_2			h_N
	s_2	s_4	s_1	s_2	s_2
s_2		s_3	s_3	s_1	s_1
s_3	s_1		s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

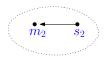
	s_1	s_2	s_3	s_4
	h	h	ℓ	l
	m	m	m	m
	ℓ	ℓ	h	h
status quo:	h_1, ℓ_1	m_2	m_3, h_3	Ø
current:	h_1, ℓ_1	m_2	m_3, h_3	Ø











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h_1	ℓ_1	m_2	h_3	m_3	h_N
s_4	32	s_4	s_1		s_2
s_2		s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

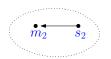
 S_1

	s_1	s_2	s_3	s_4
	h	h	l	l
	m	[m]	m	m
	ℓ	$\overline{\ell}$	h	h
status quo:	$h_1, \underline{\ell_1}$	m_2	m_3, h_3	Ø
current:	h_1,ℓ_1	m_2	m_3, h_3	Ø
		h3	• <i>s</i> ₃	v_3



 h_1

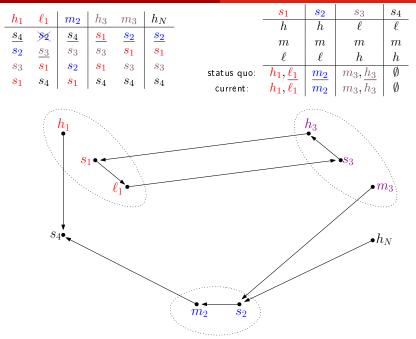




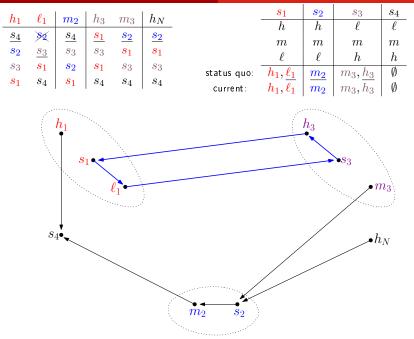
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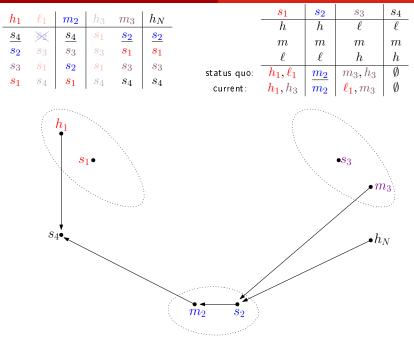
SI-CC mechanism and its properties



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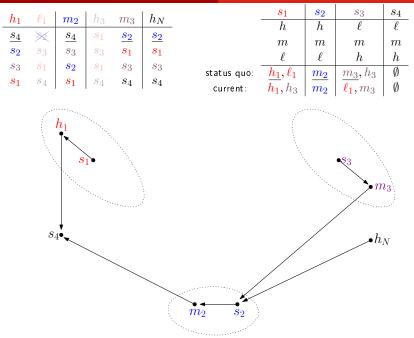


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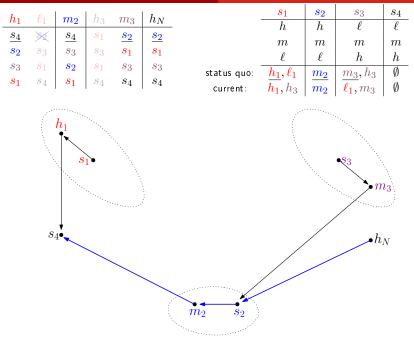
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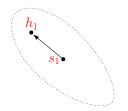
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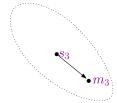


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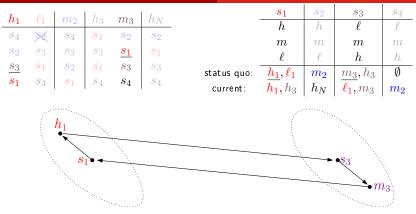
h_1	ℓ_1	m_2	h_3	m_3	h_N
s_4	×	s_4	s_1	s_2	s_2
s_2	s_3	s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	s_4	s_1	s_4	s_4	s_4

	s_1	s_2	s_3	s_4
	h	h	l	l
	m	m	m	m
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status quo:	h_1, ℓ_1	m_2	m_3, h_3	Ø
current:	$\overline{h_1}, h_3$	h_N	$\frac{m_3}{\ell_1}, m_3$	m_2
		· · · · · · · · · · · ·	1	



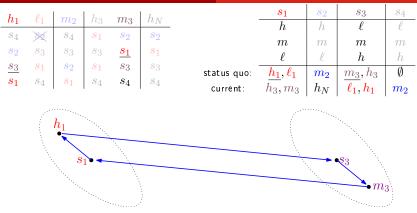


SI-CC mechanism and its properties



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SI-CC mechanism and its properties



				m_3	
s_4	×	S_4	s_1	\$2 \$1 \$3 \$4	s_2
s_2	s_3	s_3	s_3	s_1	s_1
s_3	s_1	s_2	s_1	s_3	s_3
s_1	S_4	s_1	S_4	S4	S4

	s_1	s_2	s_3	s_4
	h	h	l	l
	m	m	m	m
	l	l	h	h
status quo:	h_1, ℓ_1	m_2	m_3, h_3	Ø
current:	h_3, m_3	h_N	$m_3, h_3 \ \ell_1, h_1$	m_2

SI Cycles and Chains (SI-CC) Mechanism

School pointing rule: Pointing to lower ranked teachers first: important for strategy-proofness

Teacher pointing rule: Need a counter at each school to keep track of improvements to determine whether a teacher can point

Chain construction & selection rule: Ensure SI by not leaving occupied seats empty

Main Result

Theorem

The SI-CC mechanism is strategy-proof and constrained-efficient (IR & SI)

Remark

Any change in pointing rules in SI-CC (except tie-breaking) may lead to a violation in either SI, constrained efficiency, or strategy-proofness.

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7 Conclusion

A matching $\mu \Rightarrow$ total distrib. of experience $d^{\mu} = (d^{\mu}_s)_s$

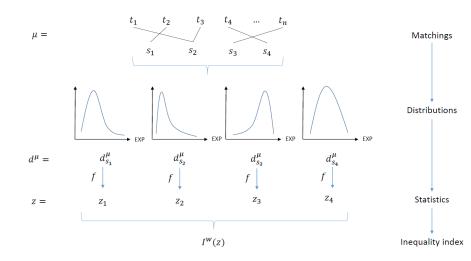
Distrib. of exp. $d^{\mu} \Rightarrow$ statistics for schools $f(d^{\mu}) \in \mathbb{R}^{m}$ Example: average experience in each school Property: must be increasing with Lorenz domination (FOSD)

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Inequality index $I(f(d^{\mu}))$: Schur-convex and cont. differentiable a.e.

Example: Gini Index

$$I(z) = \frac{1}{2\sum_{s} w_{s} z_{s}} \sum_{s} \sum_{s'} w_{s} w_{s'} |z_{s} - z_{s'}| \text{ at } z = f(d^{\mu})$$

Reducing inequalities:

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Reducing inequalities:

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Large Market and Inequality Reduction

 ν^{j} the matching of j teachers outside of the reassignment market \Rightarrow Stat. are computed using all teachers

Assumption (*f*-convergence)

For any matching μ and school $s, \ f(d^{\mu,j}) \stackrel{j \to \infty}{\longrightarrow} z^*$

 \Rightarrow The impact of the matching μ on the stat. is small when the number of outside teachers is large

E.g. in France $\approx 2\%$ ask for a reassignment

School improvement for $\triangleright^* \Rightarrow$ Inequality reduction

Proposition

Under *f*-convergence, for a large enough market size *j*, there exists priorities for schools \triangleright^* such that if μ is school improving, then μ reduces inequalities

Sketch: "increase bad schools and decrease good schools"

Partition the set of schools in two groups L and H:

$$s \in L$$
 if $\frac{\partial I}{\partial z_s}(d_s^{\omega}) < 0$

 \Rightarrow Increasing the stat. for s decreases inequalities

$$s \in L$$
 if $\frac{\partial I}{\partial z_s}(d_s^\omega) > 0$

 \Rightarrow Increasing the stat. for s increases inequalities

Priority design. define priorities \triangleright^* over types:

 $\text{ For each school } s \in L, \quad \theta_{|\Theta|-1} \, \triangleright_s^* \, \theta_{|\Theta|-2} \, \triangleright_s^* \ldots \triangleright_s^* \, \theta_1 \, \triangleright_s^* \theta_0 \\$

 \Rightarrow L schools prefer high to low exp.

For each school $s \in H$, $\theta_0 \triangleright_s^* \theta_1 \triangleright_s^* \theta_2 \triangleright_s^* \ldots \triangleright_s^* \theta_{|\Theta|-1} \triangleright_s^*$

 \Rightarrow H schools prefer low to high exp.

Sketch: "increase bad schools and decrease good schools"

Partition the set of schools in two groups L and H:

 $s \in L$ if $\frac{\partial I}{\partial z_s}(d^{\omega}) < 0$ but $\exists \mu : \frac{\partial I}{\partial z_s}(d^{\mu}) > 0$?

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$$s \in L$$
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Main problem: in general, the sign of derivatives can change ⇒ Inequality index can start to increase with SI: overshooting

In the large, with f-convergence, the sign does not change \Rightarrow Inequality index will decrease for sure with SI

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Inequality reductions \Rightarrow school improvement w.r.t. \triangleright^*

Consider a class of statistics ${\cal E}$

 \Rightarrow We want to reduce inequalities for all of them: "robustness"

Proposition (informal)

If the class \mathcal{E} is rich then for a large enough market size j, μ reduces inequalities for all $f \in \mathcal{E} \Rightarrow \mu$ statu-quo improves ω w.r.t. to \triangleright^*

Corollary (informal)

If the class \mathcal{E} is rich then for a large enough market size j, SI-CC is efficient among all mechanisms which reduce inequalities for all $f \in \mathcal{E}$

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Finite markets and SI-CC

Proposition

There is no mechanism that is strategy-proof and generates lower inequalities than SI-CC whenever possible

Related Literature

. . .

Centralized teacher (re)assignment, and other jobs:

Pereyra (2013); Combe, Tercieux, Terrier (2021); Dur & Kesten (2019); Agarwal (2015); Thakur (2020); Sidibe et al., (2021) ...

Efficient matching and constraints:

Shapley & Scarf (1974); Abdulkadiroğlu & Sönmez (1999); Papai (2000); Roth, Sönmez, Ünver (2004); Dur, Kesten, Ünver (2015); Pycia & Ünver (2017); Takamasa, Tamura, Yokoo (2018); Dur & Ünver (2019); Hafalır, Kojima, Yenmez (2022) ...

Stable/fair matching and constraints:

Gale & Shapley (1962); Kelso & Crawford (1982); Roth & Sotomayor (1990); Balinski & Sönmez (1999); Abdulkadiroğlu & Sönmez (2003); Hatfield & Milgrom (2005); Kominers & Sönmez (2016); Hafalır, Yenmez, Yıldırım (2013); Ehlers et al. (2014); Kojima & Kamada (2015, 2016); Dur et al. (2018, 2020); Sönmez & Yenmez (2022) ...

Unequal distribution of teachers across schools:

Bobba et al. (2021); Bates et al. (2021); Biasi et al. (2021); Tincani (2021)



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Empirical Analysis

Combe, Dur, Tercieux, Terrier, Ünver Better Distribution through (Re)assignme

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French teacher assignment

Data on French centralized assignment of teachers to regions in 2013

Estimation of teachers' pref over regions: $u_{t,R}$ Details

Separate estimation for tenured teachers and newcomers

Estimation on each of 8 fields (Maths, History, Sport...)

Final sample: 10,460 teachers: 5,833 tenured teachers (55.8%) and 4,627 new teachers

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Aims to quantify the algorithms performance in a real-life setting:

SI-CC (our SI constrained efficient mechanism)

Benchmark for SI-CC: TTC*

As SI-CC but does not impose status-quo improvement for schools (\sim school choice TTC with IR).

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The currently used mechanism which is a variation of Deferred Acceptance with IR.

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Teacher Types and Regions' Preferences

Teacher type:

Corresponds to her experience We classify teachers into 13 experience bins

Inequality index and schools' priorities

We use the Gini index together with the mean experience statistic in each region

We follow the priority design construction from the theory

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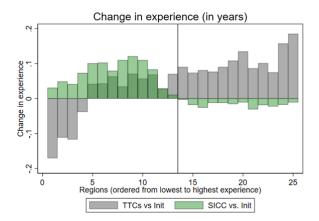
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		Regions	Teacher-Type Ranking ⊳
1	L = Gini derivative < 0	Créteil Versailles Amiens	High $exp \triangleright \ldots \triangleright$ Low $exp \triangleright \emptyset$
2	$H = {\sf Gini} \; {\sf derivative} > 0$	Bordeaux Rennes Lyon	Ø⊳ Low exp ⊳ ⊳ High exp

Empirical results: SICC decreases inequalities

	Statu-quo	SI-CC	TTC*	Current French			
	(1)	(2)	(3)	(4)			
	Panel A. Teacher mobility						
Mobility	-	5,221	6,393	5,866			
From L regions	-	798	1,843	1,547			
From H regions	-	512	639	407			
Tenured	-	1,310	2,481	1,954			
Newcomers	-	3,912	3,912	3,912			
Number of unassigned teachers	-	715	715	715			
Panel B. Average rank of region obtained							
Average rank of assigned region	16,28	7.58	7.31	8.27			
Teachers from the L regions	8.3	7.28	5.67	6.15			
Teachers from the H regions	9.4	4.71	3	5.39			
Tenured	8.47	6.91	5.29	6.04			
Newcomers	26	8.42	9.85	11.08			
Panel C. Inequalities							
Gini index	0.048	0.047	0.051	0.051			
Average exp. in L regions	11.98	12.05	11.94	11.96			
Average exp. in H regions	14.01	13.99	14.11	14.10			

Empirical results: SICC does decrease inequalities







We design schools' priorities that reflect a central authority's welfare objective, here: reduce inequalities

We design a mechanism SI-CC that improve both teachers welfare and schools' welfare (proxy to reduce ineq.) compared to an initial allocation.

Our counterfactual analysis using French data shows that

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Appendix

Combe, Dur, Tercieux, Terrier, Ünver Better Distribution through (Re)assignme

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Step k:

Each remaining school s points to its remaining least preferred statsus-quo employee (if there are many, it uses a fixed tie-breaker). Each remaining teacher t points to her top choice among \emptyset and all remaining schools s that satisfy:

Type (1) School improvement by replacement:

if s points to a teacher t^\prime and replacing her with t will make s weakly better off than its status quo assignment,

or

Type (2) School improvement by addition: if t is acceptable for s and s has a vacant seat.

 \emptyset points to all teachers pointing to it.

Step k continued:

Two cases:

 (i) There exists a cycle in which either all teachers' pointing satisfies (1) or there are only one teacher and option Ø

Each teacher is assigned to the school/option she is pointing to, go to Step k+1.

(ii) There exists a chain and (i) does not hold.

- If there is a remaining new teacher: we select a starting with the new teacher and ending with a school with a vacant position
- Otherwise: we remove each school s whose all status-quo employees are assigned, go to Step k+1.

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Demand estimation

Random utility model estimated as a function of teachers' and regions' characteristics

$$u_{t,R} = \delta_R + Z'_{t,R}\beta + \varepsilon_{t,R} \tag{1}$$

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 $\begin{array}{ll} \delta_R & \mbox{region fixed effect} \\ Z_{t,R} & \mbox{teacher-region-specific observables} \\ \varepsilon_{t,R} & \mbox{random shock i.i.d. over } t \mbox{ and } R \\ & \mbox{type-l extreme value distribution, Gumbel(0,1)} \end{array}$

Goal: Estimate the model and run counter-factuals

- Separate estimation for tenured teachers and newcomers
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Demand estimation

Teacher characteristics:

Qualification

Experience

Family status

Teacher-region specific characteristics:

Birth region

. . .

Current region

(Interacted) Region characteristics:

Socio-economic measure

Academic performance measure

Preference estimation

Identifying assumption based on stability

(Chiappori-Salanié (16), Akyol-Krishna (17), Artemov-Che-He (19), Fack-Grenet-He (19))

Teachers might skip unreachable regions from their ranking **but** Assignment = most preferred region within <u>feasible</u> regions

Logit choice probabilities. Estimate β via ML.

Fit is a lot better than assuming truthtelling (Back).

Appendix

Eight markets

			Initially	Vacant
	All teachers	Newcomers	assigned	positions
	(1)	(2)	(3)	(4)
All subjects	10460	4627	5833	3912
Sport	2066	568	1498	475
French	1645	786	859	663
English	1374	746	628	640
Mathematics	1563	958	605	824
Spanish	999	316	683	248
History-Geography	1230	657	573	562
Biology	746	286	460	246
Physics-Chemistry	837	310	527	254

Teachers' characteristics

	Tenured			Newcomers		
	French (1)	Math (2)	English (3)	French (4)	Math (5)	English (6)
% Female	76.1	47.0	85.4	80.3	41.7	80.4
% Maried	48.5	45.0	46.8	41.1	39.4	40.9
% In disadvantaged school	10.4	13.2	4.4	0.0	0.0	0.0
Experience (in years)	7.48	7.23	7.18	2.76	2.24	2.30
% Advanced teaching qualif	7.9	29.1	8.8	16.8	31.7	15.2
Observations	859	605	628	786	958	746

