

Title: From dg-categories to quasi-categories in modules

Abstract:

In his seminal 2004 paper "The homotopy theory of dg categories and derived Morita theory", Bertrand Toën studies the simplicial localisation of  $\text{dgcats}$  at the quasi-equivalences and establishes an inner  $\text{RHom}$ , making the homotopy category  $\text{hoddgcats}$  into a monoidal closed category in spite of  $\text{dgcats}$  not being model monoidal for the Tabuada model structure. Drawing from the analogy between dg-categories and simplicial categories, several attempts have been undertaken at introducing weak versions of dg-categories that should allow for a monoidal model structure, like the dg-Segal categories of Hugo Bacard (2010) and, more recently, the dg-Segal spaces of Elena Dimitriadis-Bermejo (2022). In this talk, we focus on the templicial modules introduced by Arne Mertens in his 2022 PhD thesis as a candidate framework to model weak enrichment in simplicial modules. In particular, we describe an enrichment of the dg-nerve landing in quasi-categories in modules. Finally, we discuss joint work with Violeta Borges Marques and Arne Mertens showing that quasi-categories in modules are stable under infinitesimal deformation.