

Zhu, Chenchang

Title: Shifted symplectic structures on higher groupoids and higher cotangent bundles

Abstract: In this talk, we will first shortly describe a way to realise shifted symplectic structure in differential stacks via Lie  $n$ -groupoids. As we know, one of the key ingredients in symplectic dynamical system and classical mechanics is the natural symplectic structure  $\omega_c$  on the cotangent bundle  $T^*M$  of a manifold  $M$ . Many natural symplectic structures come from the symplectic reduction of  $(T^*M, \omega_c)$  as reduced phase spaces. When we go higher, PTVV shows that the shifted cotangent bundle of a derived higher stack carries a shifted symplectic structure. In contrast to algebraic geometry, where a cotangent bundle can be simply defined as spectrum of symmetric algebra of tangent sheaves, the concept of cotangent bundle of a higher Lie groupoid as higher VB (vector bundle) groupoid can not be directly obtained. There are many Lie  $n$ -groupoids presenting the same  $n$ -stack. But we believe there is a canonical VB Lie  $n$ -groupoid presenting the cotangent bundle as soon as we choose a fixed Lie  $n$ -groupoid  $X_\bullet$  presenting the  $n$ -stack, just as the tangent bundle is canonically given by the tangent VB  $n$ -groupoid  $TX_\bullet$ . Notice that the naive choice  $T^*X_\bullet$  does not make sense at all: even for  $n=1$ , the cotangent groupoid of a Lie groupoid  $G:=G_1 \rightrightarrows G_0$  is  $T^*G_1 \rightrightarrows A^*$ , where  $A^*$  is the Lie algebroid of  $G$ . The missing of explicit expression of the higher cotangent bundle blocks us from further developing higher and derived structures in dynamical system and mechanics. Thus in the second part of the talk, we will present a way to give explicit formula of cotangent bundle for Lie 2-groupoid and sketch some applications, including integration of general Courant algebroids and higher moment maps.