Symplectic monodromy at radius zero

by Tomasz Pełka (University of Warsaw)

The Zariski multiplicity conjecture asserts that a family of isolated hypersurface singularities with constant Milnor number has constant multiplicity. The aim of this course is to explain the key tool used in its proof – the A'Campo space – and outline a range of its possible applications.

The proof of the Zariski conjecture is inspired by a result of McLean, interpreting the multiplicity – an algebraic invariant – in terms of symplectic geometry: more precisely, in terms of Floer homology of the monodromy of Milnor fibration. In order to effectively apply Floer theoretic tools from McLean's proof, we provide an explicit model of this monodromy, with good dynamical properties. Its underlying topological space was constructed by A'Campo, we endow it with a very particular symplectic structure.

This construction allows to transport – via symplectic connection – complicated geometry of Milnor fibration in the tube of positive radius to *radius zero*, where tropical coordinates make the picture essentially combinatorial. This explicit hybrid construction can be applied in a variety of other contexts. For example, at radius zero one readily sees Lagrangian tori, where the monodromy is simply a translation. Once pushed back to the Milnor fiber, those tori behave surprisingly well: for example, in the context of maximal Calabi–Yau degenerations, they give some approximation of special Lagrangian torus fibration expected by Mirror Symmetry.

This is a joint work with J. F. de Bobadilla.