Optimal Binary Hierarchical Poset Code Having Hull Dimension One

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Outline

- Introduction
- Hull of Hierarchical Poset Code
- Construction
- Key Results
- Conclusion
- References

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- ⋆ Partial order over finite set of n elements is defined on $X = \{1, 2, \cdots, n\}$ upto isomorphism.
- ⋆ A subposet $I \subseteq X$ is called an *ideal* of P if it satisfies the closeness property : if $i \in I$ and $j \preceq i$, then $j \in I$.

Definition 1

Let $P = ([n], \preceq)$ be a poset on $[n]$. Given a vector $x = (x_1, x_2, \cdots, x_n) \in \mathbb{F}_q^n$, *P-weight* is defined by $\varpi_P(x) = |\langle supp(x) \rangle|$.
where, $supp(x) = \{i : x_i \neq 0\}$

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Definition 2

P-distance between any two vector y, z belong to $\mathbb{F}_q^n$ is defined as $d_P(y, z) = \varpi_P(y - z)$. This distance satisfies all the properties of metric so it is known as **Poset metric**.

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Poset Metrics : The State of the art

Definition 3

If $\mathbb{F}_q^n$ endowed with a poset metric then a subspace C of $\mathbb{F}_q^n$ called a poset code.

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- Poset metric is the generalization of Hamming metric and the Niederreiter -Rosenbloom Tsfasman metric.
- General poset metric gave a new approach towards several classical invariants of coding theory (like Minimum distance, Packing radius, Covering radius) and most of its fundamental properties (such as Perfect code, MDS code, MacWilliams identities etc.).

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Definition 6

The i^{th} level H_i of a poset P :

$$H_i = \{b \in [n] : h(b) = i\}$$

Introduction

Definition 7

Consider a partition
$$X = \dot{\bigcup}_{i=1,2,\ldots,l} H_i$$
with $h_i = |H_i| > 0$ and $n = h_1 + h_2 + ... + h_l$. where $\dot{\cup}$ denotes the union of disjoint sets. Define $H = (H_1, H_2, ..., H_l)$ and $h = (h_1, h_2, ..., h_l)$ to be hierarchy spectrum and hierarchical array, respectively. A hierarchical poset with hierarchy spectrum H is the poset $P_H = ([n], \preceq_H)$ where
$$a \preceq_H b \; \textit{if and only if} \quad a \in H_i, b \in H_j \quad \textit{and } i < j.$$

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$$a \preceq_H b \; if\; and\; only\; if \quad a \in H_i, b \in H_j \quad and\; i < j.$$

Definition 8

Consider the hierarchical poset $P_H = ([n], \preceq_H)$ with hierarchy spectrum H. Given $x \in \mathbb{F}_q^n$ we write $x = x_1 + x_2 + \ldots + x_l$ where $supp(x_i) \subseteq H_i$ then if $M(x) = max\{i : x_i \neq 0\}$. We have that $\langle supp(x) \rangle = (supp(x) \cap H_{M(x)}) \dot{\bigcup} (\bigcup_{i=1}^{M(x)-1} H_i)$ and the disjoint union ensures that

$$w_H(x) = |supp(x^{M(x)})| + \sum_{i=1}^{M(x)-1} h_i \tag{1}$$

where $(h_1, h_2, \ldots, h_l)$ is the hierarchical array of H.

Definition 9

Hierarchical poset distance between any two vector y, z belong to $\mathbb{F}_q^n$ is defined as $d_H(y,z) = w_H(y-z)$. This distance satisfies all the properties of metric so it is known as **Hierarchical Poset metric**.

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Definition 10

A linear code $C \subseteq \mathbb{F}_q^n$ is called an hierarchical poset code when we consider on $\mathbb{F}_q^n$ a metric determined by an hierarchical poset.

Theorem 11

*The poset $P = ([n], \preceq)$ having l levels is hierarchical if, and only if, any linear code $C \subseteq \mathbb{F}_q^n$ satisfies a P-canonical decomposition [**R3**].*

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Theorem 12

Consider $P = ([n], \preceq_{\mathcal{H}})$ be a hierarchical poset having l levels and denote $n_i = |H_i|$. Let $\{0\} \neq C \subseteq \mathbb{F}_q^n$ be a linear code and $C_1 \oplus C_2 \oplus \cdots \oplus C_l$ a P-canonical decomposition of C. Then :

$$d_P(C) = \sum_{i=1}^{t_1 - 1} n_i + d_H(C_{t_1}) \tag{2}$$

*where $t_1 = min\{i \in [l] : C_i \neq 0\}$ and $d_H(C_{t_1})$ is the minimum distance of C_{t_1} considered as a code in the Hamming space $\mathbb{F}_q^{n_{t_1}}$ [**R3**].*

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- The hull of linear codes with low dimensions gets much interest due to its crucial role in determining the complexity of algorithms for computing the automorphism group of a linear code [**R5**] and checking permutation equivalence of two linear codes [**R6**].
- The Euclidean hull of a linear code has been applied to entanglement-assisted quantum error-correcting codes (EAQECCs) via classical error-correcting codes [**R7**].

Theorem 13 (Griesmer Bound :)

Consider q be a prime power, if there exist a linear code with parameter $[n, k, d]$ *then*

$$n \geq \sum_{i=0}^{k-1} \left\lceil \frac{d}{q^i} \right\rceil \tag{3}$$

Hull Of Hierarchical Poset Code

Let C be a hierarchical poset code over $\mathbb{F}_q$ having length n, then its dual is defined as

$$C^{\perp} = \{v' \in F_q^n : \langle v', u' \rangle = 0 \ \ \forall u' \in C\}$$

where $\langle ., . \rangle$ is a inner product on $\mathbb{F}_q^n$ such as the Euclidean inner product.
The **Hull of hierarchical poset code** is defined as $Hull(C) = C \cap C^{\perp}$.

Theorem 14

Let a hierarchical poset code C with parameter $[n, k, d_{\mathcal{H}}]$ having generator matrix G. Then,

$$dim(Hull(C)) = k - rank(GG^t) \tag{4}$$

Construction

We have assumed that hierarchical poset of l level i.e. $P(n; n_1, n_2, \cdots, n_l)$ with $n = n_1 + n_2 + \cdots + n_l$. Consider a binary hierarchical poset code C whose basis elements are $\beta_\mathbf{1}$ and $\beta_\mathbf{2}$ which are constructed as below :

$$\beta_\mathbf{1} = [b_{11}, b_{12}, \cdots, b_{1n_1}, b_{1(n_1+1)}, \cdots, b_{1(n_1+n_2)}, \cdots, b_{1n}] = [\mathbf{u_{11}}, \mathbf{u_{12}}, \cdots, \mathbf{u_{1l}}]$$
$$and \quad \beta_\mathbf{2} = [b_{21}, b_{22}, \cdots, b_{2n_1}, b_{2(n_1+1)}, \cdots, b_{2(n_1+n_2)}, \cdots, b_{2n}] = [\mathbf{u_{21}}, \mathbf{u_{22}}, \cdots, \mathbf{u_{2l}}] \tag{5}$$

Where,

$$\mathbf{u_{1i}} = \left[b_{1((\sum_{k=1}^{i-1} n_k)+1)}, \cdots, b_{1(\sum_{k=1}^{i} n_k)} \right]$$

and

$$\mathbf{u_{2i}} = \left[b_{2((\sum_{k=1}^{i-1} n_k)+1)}, \cdots, b_{2(\sum_{k=1}^{i} n_k)} \right] \tag{6}$$

Let, $y_1^{\beta_1} = min\{i \in [l] : u_{1i} \neq 0\}$ and $y_1^{\beta_2} = min\{i \in [l] : u_{2i} \neq 0\}$.
Let, $y_1 = min\{y_1^{\beta_1}, y_1^{\beta_2}\}$

By using Theorem 11 we can rewrite C as follows : $C = \bar{C}_1 \oplus \bar{C}_2 \oplus \cdots \oplus \bar{C}_l$ where $supp(\bar{C}_i) \subseteq H_i$. By above construction, we obtained $\bar{C}_i = \{0\}$ for $i = 1, 2, \cdots, y_1 - 1$ By using Theorem12 and construction we can calculate minimum distance of C which as follows :

$$d_P(C) = \sum_{i=1}^{y_1-1} n_i + d_H(\bar{C}_{y_1}) \tag{7}$$

By the construction of $[n, 2]$ hierarchical poset code generator matrix can be given by

$$G = \begin{bmatrix} b_{11} & b_{12} & \cdots b_{1n_1} & b_{1(n_1+1)} & \cdots & b_{1n} \\ b_{21} & b_{22} & \cdots b_{2n_1} & b_{2(n_1+1)} & \cdots & b_{2n} \end{bmatrix} \tag{8}$$

We have,

$$G = \begin{bmatrix} \boldsymbol{\beta_1} \\ \boldsymbol{\beta_2} \end{bmatrix} \tag{9}$$

where,

$$\boldsymbol{\beta_1} = \begin{bmatrix} b_{11} & \cdots & b_{1(n_1+1)} & \cdots b_{1n} \end{bmatrix} \textit{and} \;\; \boldsymbol{\beta_2} = \begin{bmatrix} b_{21} & \cdots & b_{2(n_1+1)} & \cdots b_{2n} \end{bmatrix} \tag{10.1}$$

Alternatively, by setting

$$\boldsymbol{\alpha_\lambda} = \begin{bmatrix} b_{1\lambda} \\ b_{2\lambda} \end{bmatrix}$$

For $1 \leq \lambda \leq n$, **G** can be written as follows :

$$\mathbf{G} = \begin{bmatrix} \boldsymbol{\alpha_1} & \boldsymbol{\alpha_2} & \cdots & \boldsymbol{\alpha_n} \end{bmatrix}$$

For, $i, j \in F_2$, Consider,

$$\boldsymbol{R_{ij}} = \left| \left\{ \lambda \in \{1, 2, \cdots, n\} | \begin{bmatrix} b_{1\lambda} \\ b_{2\lambda} \end{bmatrix} = \begin{bmatrix} i \\ j \end{bmatrix} \right\} \right|$$

Also,

$$\boldsymbol{R_{ij1}} = \left| \left\{ \lambda \in \{1, 2, \cdots, \sum_{i=1}^{y_1} n_i\} | \begin{bmatrix} b_{1\lambda} \\ b_{2\lambda} \end{bmatrix} = \begin{bmatrix} i \\ j \end{bmatrix} \right\} \right|$$

and

$$\boldsymbol{R_{ij2}} = \left| \left\{ \lambda \in \{\sum_{i=1}^{y_1} n_i + 1, \cdots, n\} | \begin{bmatrix} b_{1\lambda} \\ b_{2\lambda} \end{bmatrix} = \begin{bmatrix} i \\ j \end{bmatrix} \right\} \right|$$

$$\implies \boldsymbol{R_{ij}} = \boldsymbol{R_{ij1}} + \boldsymbol{R_{ij2}}$$

By above construction of C, we have ;

$$\boldsymbol{R_{00}} = \sum_{i=1}^{y_1-1} n_i \tag{11}$$

From Theorem12, We have,

$$d_P(C) = R_{00} + min\{R_{101} + R_{111}, R_{011} + R_{111}, R_{101} + R_{011}\} \tag{12}$$

$$GG^t = \begin{bmatrix} R_{10} + R_{11} & R_{11} \\ R_{11} & R_{01} + R_{11} \end{bmatrix} (mod \;\; 2) \tag{13}$$

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Definition 15

For prime power q, $h_d \in \mathbb{Z}^+$ and positive integer n, k. Let,
$A_q(n, k, h_d) = \max\left\{ d \mid \exists \text{ hierarchical poset code having hull dimension } h_d \right\}$

Theorem 16

Let q be a prime power, h_d be a non-negative integer and n, k be integers such that $1 \leq k \leq n$. Then,

$$A_q(n, k, h_d) \leq \left\lfloor \frac{(q-1)q^{k-1} n_{y_1}}{(q^k - 1)} \right\rfloor + \sum_{i=1}^{y_1 - 1} n_i \quad (14)$$

Proof Outline.

This theorem is proved by using Theorem13 and Theorem12. □

Key Results

Theorem 17

If $n \equiv 3 \pmod 9$,

$$\sum_{i=1}^{y_1-1} n_i = R_{00} = v, \sum_{i=y_1+1}^{l} n_i = 2v = R_{102} + R_{112} + R_{012}, n_{y_1} = 6v + 3,$$

then $A_2(n,2,1) = \lfloor 5v + 2 \rfloor$ *and there exist a unique (upto equivalence) optimal* $[n, 2, \lfloor 5\upsilon + 2 \rfloor]$ *code with hull dimension one by considering hierarchical poset of level one for all* $n > 3$.

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then $A_2(n, 2, 1) = \lfloor 5v + 2 \rfloor$ and there exist a unique (upto equivalence) optimal $[n, 2, \lfloor 5\upsilon + 2 \rfloor]$ code with hull dimension one by considering hierarchical poset of level one for all $n > 3$.

Proof Outline :

The existence of an hierarchical poset code having parameters $[n, 2, 5v + 2]_2$ is proved by using $(R_{101}, R_{111}, R_{011}, R_{102}, R_{112}, R_{012}) = (2\upsilon + 1, 2\upsilon + 1, 2\upsilon + 1, 2\upsilon - 1, 1, 0)$.
In case of hierarchical poset of level one, the enumeration of codes is given upto equivalence.
To determine the number of $[n, 2, 5\upsilon + 2]_2$ codes with hull dimension one, we claim that $R_{101} + R_{111} + R_{011} + R_{102} + R_{112} + R_{012} + R_{00} = n$.
By verifying the possibilities of $(R_{101}, R_{111}, R_{011}, R_{102}, R_{112}, R_{012})$, we conclude that there exist a unique (upto equivalence) optimal $[n, 2, \lfloor 5\upsilon + 2 \rfloor]$ code with hull dimension one under hierarchical poset of level one for all $n > 3$. □

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Key Results

Theorem 18

Suppose $n \geq 3$ be a natural number. If $n \equiv 5$ (mod 9),

$$\sum_{i=1}^{y_1-1} n_i = R_{00} = u, \sum_{i=y_1+1}^{l} n_i = 2u = R_{102} + R_{112} + R_{012}, n_{y_1} = 6u + 5,$$

then $A_2(n, 2, 1) = \lfloor 5u + 3 \rfloor$. In this case, by considering anti-chain, there exist a unique (upto equivalence) optimal $[5, 2, 3]_2$ code with hul dimension one. and there are three (up to equivalence) optimal $[n, 2, \lfloor 5u + 3 \rfloor]$ code with hull dimension one for all $n > 5$.

Proof Outline :

To establish the existence, it is sufficient to demonstrate that there exist an $[n, 2, 5u + 3]_2$ code having hull dimension one, which is achieved by using $(R_{101}, R_{111}, R_{011}, R_{102}, R_{112}, R_{012}) = (2u + 2, 2u + 1, 2u + 2, 2u, 0, 0)$.
In case of anti-chain poset, to determine the number of $[n, 2, 5u + 3]_2$ codes with hull dimension one, we claim that $R_{101} + R_{111} + R_{011} + R_{102} + R_{112} + R_{012} + R_{00} = n$.
By checking the possibilities of $(R_{101}, R_{111}, R_{011}, R_{102}, R_{112}, R_{012})$, we obtained three families of codes upto equivalence and under some condition uniqueness is obtained. □

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Key Results

Note :

Following similar arguments, we have established results for $n \equiv \zeta (mod \ \ 9)$, where, $\zeta = 1, 2, 4, 6, 7,$ and 8 with condition mentioned in following table.

$n \equiv \zeta$ $(mod\ 9)$	$\sum_{i=1}^{y_1-1} n_i = R_{00}$	$\sum_{i=y_1+1}^{l} n_i = R_{102}+R_{112}+R_{012}$	n_{y_1}	$A_2(n,2,1)$
$\zeta = 0$	$a-3$	$2a$	$6a+3$	$\lfloor 5a-1 \rfloor$
$\zeta = 1$	a	$2a$	$6a+1$	$\lfloor 5a \rfloor$
$\zeta = 2$	$s+1$	$2s$	$6s+1$	$\lfloor 5s+1 \rfloor$
$\zeta = 4$	$t+3$	$2t$	$6t+1$	$\lfloor 5t+3 \rfloor$
$\zeta = 6$	$r+5$	$2r$	$6r+1$	$\lfloor 5r+5 \rfloor$
$\zeta = 7$	w	$2w$	$6w+7$	$\lfloor 5w+4 \rfloor$
$\zeta = 8$	$b+7$	$2b$	$6b+1$	$\lfloor 5b+7 \rfloor$

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Conclusion

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- Proved that the bound for distance of hierarchical poset code.
- Proved existence and uniqueness of Optimal Binary Hierarchical Poset Code With Hull Dimension One.

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Thank You !