Lengths of divisible codes

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(Joint work with Sascha Kurz)

A linear code C over $\mathbb{F}_q$ is called Δ-divisible if the Hamming weights $\mathrm{wt}(c)$ of all codewords $c \in C$ are divisible by Δ. The study of divisible codes was initiated by Harold Ward [1]. Linear codes meeting the Griesmer bound in many cases have to admit a relatively large divisibility constant Δ, see [2]. In order to state a connection between divisible codes and Galois geometries we associate each subspace $U \in \mathrm{PG}(n-1,q)$ with its characteristic function χ_u mapping each point $\mathrm{PG}(n-1,q)$ to an non-negative integer multiplicity, i.e., $\chi_U(P) = 1$ iff $P \le U$ and $\chi_U(P) = 0$ otherwise. We say that a mapping χ from the pointset of $\mathrm{PG}(n-1,q)$ to $\mathbb{N}$ is Δ-divisible if the corresponding linear code C_χ associated with the multiset of points characterized by χ is Δ-divisible.

Lemma *([3, Lemma 11])*
Let $\mathcal{U}$ be a multiset of subspaces of $\mathrm{PG}(n-1,q)$ with dimension at least k. Then $\chi_{\mathcal{U}} := \sum_{U \in \mathcal{U}} \chi_U$ is q^{k-1}-divisible.

If χ is Δ-divisible and $\chi(P) \le \lambda$ for some constant $\lambda \in \mathbb{N}$ and all points P, then also the λ-complement $\overline{\chi}$, defined by $\overline{\chi}(P) = \lambda - \chi(P)$ for all points P, is also Δ-divisible. The possible effective lengths of q^r-divisible codes have been completely characterized for each prime power q and each non-negative integer r in [3]. An implication of these results is that each set S of pairwise disjoint k-dimensional subspaces in $\mathrm{PG}(n-1,q)$ satisfies $\#S \le \left(q^n - q^{k+r}\right) / \left(q^k - 1\right) + 1$ if $k > (q^r - 1)/(q-1)$, where $n = tk + r$ with $r \in \{1, \ldots, k-1\}$. This upper bound for partial spreads can indeed be attained and was initially proven in [5]. Other upper bounds for partial spreads, see e.g. [4], additionally use the fact that the corresponding divisible linear code has to be projective.

More and more applications of divisible codes emerged in the last years, e.g. upper bounds for so-called subspace codes. Noting that the characterization result for the possible (effective) lengths of q^r-divisible codes from [3] involves quite large point multiplicities on the constructive side, there is quite some need for more refined results taking other parameters like the maximum possible point multiplicities or the dimension. Also the restriction that the exponent r in the divisibility constant $\Delta = q^r$ has to be an integer is not always met in the applications. In this talk I present some partial results on the possible effective lengths of divisible codes with extra constraints.

References

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