Weierstrass semigroups at the $\mathbb{F}_{q^2}$-rational points of a maximal curve with the third largest genus

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Abstract

An $\mathbb{F}_{q^2}$-maximal curve X of genus g is defined to be a projective, geometrically irreducible, non-singular algebraic curve defined over $\mathbb{F}_{q^2}$ such that the number of its $\mathbb{F}_{q^2}$-rational points attains the Hasse-Weil upper bound. $\mathbb{F}_{q^2}$-maximal curves, especially those with large genus, are of particular interest in coding theory since they give rise to excellent AG codes. It is well known that, for an $\mathbb{F}_{q^2}$-maximal curve X, $g(X) \leq q(q-1)/2$ and that it reaches this upper bound if and only if X is $\mathbb{F}_{q^2}$-isomorphic to the Hermitian curve. The first and the second largest genera of $\mathbb{F}_{q^2}$-maximal curves are known, and they are realized by exactly one curve up to $\mathbb{F}_{q^2}$-isomorphism. The value of the third largest genus is known to be equal to $g_3 = \lfloor (q^2 - q + 4)/6 \rfloor$, but it is still unclear whether this is realized by exactly one curve up to $\mathbb{F}_{q^2}$-isomorphism. In this talk, I will present our results on the Weierstrass semigroups at the $\mathbb{F}_{q^2}$-rational points of the curve $X_3 : x^{(q+1)/3} + x^{2(q+1)/3} + y^{q+1} = 0$, with $q \equiv 2 \pmod 3$, which is a curve known to have genus equal to g_3. One of the surprising results is that there are roughly $(q+1)/3$ possible different semigroups, although not all of them may occur for a given q. Moreover, the curve X_3 has many non-$\mathbb{F}_{q^2}$-rational Weierstrass points.

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