Lower bound on the maximal number of rational points on curves over finite fields

Elisa Lorenzo García
University of Neuchatel

Abstract

For a long time people have been interested in finding and constructing curves over finite fields with many points. For genus 1 and genus 2 curves, we know how to construct curves over any finite field of defect less than 1 or 3 (respectively), i.e. with a number of points at distance at most 1 or 3 to the upper bound given by the Hasse–Weil–Serre bound. The case of genus 3 is still open after more than 40 years of research. In this talk I will take a different approach based on the random matrix theory of Katz–Sarnak, that describe the distribution of the number of points, to prove the existence, for all $\epsilon > 0$, of curves of genus g over $\mathbb{F}_q$ with more than $1+q+(2g-\epsilon)\sqrt{q}$ points for q big enough. I will also discuss some explicit constructions as well as some details about the asymmetric of the distribution of the trace of the Frobenius for curves of genus 3.

This is a joint work with J. Bergström, E. Howe and C. Ritzenthaler.