Elliptic curves: parity phenomena

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(Mostly a survey, with some joint work with L. Cowland Kellock.)

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All $n\in\mathbb{Z}[i]\setminus\{0\}$ can be written as $n=s^2k(t)$ for some $s,t\in\mathbb{Q}(i)$, where $k(t)=t^3-\frac{3}{4}t^2-2t-1$.

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$y^2+y=x^3-x$ acquires new solutions over $K(\sqrt{\Delta_K})$, provided $\sqrt{\Delta_K} \notin K$ and $37 \nmid \Delta_K$.

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Fix $K/\mathbb{Q}$ Galois with Galois group $D_{2q} \times D_{2r} \times D_{2s} \times D_{2t}$ for distinct primes $q, r, s, t \geq 5$. For all $E/\mathbb{Q}$, each 16-dim. irred. representation of $\mathrm{Gal}(K/\mathbb{Q})$ has even multiplicity in $E(K) \otimes_{\mathbb{Z}} \mathbb{C}$.

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$E : y^2 + y = x^3 - x$ Galois $F/\mathbb{Q}$ with $\mathrm{Gal}(F/\mathbb{Q}) \simeq A_5$ and $37 \nmid \Delta_F$.

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W_G = direct sum of the selfdual odd-dimensional irreducible representations of a group G.

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If $F/\mathbb{Q}$ Galois with no quadratic subfields, then for 100% elliptic curves $E/\mathbb{Q}$ with $(\Delta_E,\Delta_F)=1$, $E(F)\otimes_{\mathbb{Z}}\mathbb{C}$ is either $W_{\mathrm{Gal}(F/\mathbb{Q})}$ or 0.

Thank you!