

# Perfectoid Hilbert-Kunz multiplicity

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The Hilbert-Samuel multiplicity is a classical invariant that measures the singularities of local rings. In positive characteristic, via the Frobenius map, Monsky has introduced the Hilbert-Kunz multiplicity, which is a more subtle invariant and is a finer measure of the singularities. In this talk, we introduce and study a mixed characteristic version of the Hilbert-Kunz multiplicity using Faltings' normalized length function and Bhatt-Scholze's perfectoidization functor. We show that this perfectoid Hilbert-Kunz multiplicity shares many common properties as the Hilbert-Kunz multiplicity, for example it gives a criterion for regular local rings, and characterizes extended plus closure of  $\mathfrak{m}$ -primary ideals. We also raise some interesting questions. All results in mixed characteristics are in joint work with Hanlin Cai, Seungsu Lee, Karl Schwede, and Kevin Tucker. Our presentation has close connections (and also serves as a preliminary talk) to Tucker's talk on perfectoid signature.