Quasi-stationary distributions without killing

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QSDs are crucial objects for the understanding of metastable systems. In the context of finite states Markov chain, QSDs can be characterized algebraically as the principal left eigenfunctions of the sub-markovian kernel obtained by imposing a killing condition (usually, considering the boundary of the region as absorbing). An alternative, probabilistic, construction is available in terms of the so-called Yaglom limit, i.e., the limiting distribution of the process conditioning on the event in which the chain has not being killed. From the physical point of view, one might be tempted to describe QSDs as *apparent equilibria of a metastable systems*, i.e., states in which the system settles on timescales smaller than the one in which the equilibrium is reached. In this sense, the *killing time* could be rephrased by a certain notion of *mixing time*. In this spirit, we provide an analogue notion of QSD which does not require killing, and can be read both in terms of the eigenfunctions of the underlyinh process and by a limiting distribution procedure similar to the Yaglom limit. We also show that such notion of QSD enojoys the crucial *exponential exit time* property.

The talk is based on a ongoing work with R. Fernandez, F. Manzo and E. Scoppola