Large mean-field systems conditioned by rare events and link to stochastic control

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We study the limit behavior of a large mean-field system of interacting diffusion processes, while conditioning a symmetric function of the particles to stay bounded at each time. As the number of particles goes to infinity, this amounts to conditioning by a zero-probability event. The limit is obtained using large deviation methods on the path space to prove a Gibbs conditioning principle for mean-field systems. The limit process is then characterized as the optimiser of a McKean-Vlasov control problem with constraints on the law of the controlled process. The related optimality conditions provide a forward-backward system involving a non-linear Fokker-Planck equation coupled to a Hamilton-Jacobi-Bellman equation.

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