Path large deviations for kinetic theories: beyond the Boltzmann, the Landau, the Balescu–Lenard–Guernsey, and the weak turbulence kinetic equations
Rare event dynamics applied to climate models

Freddy Bouchet

March 22, 2023

Joint works with Gregory Eyink, Ouassim Feliachi and Jules Guioth

In many physical systems one seeks to describe effectively mesoscopic or macroscopic variables. Kinetic theories and kinetic equations are examples where the average mesoscopic dynamics is obtained through very clear theoretical procedures and can possibly lead to mathematical proofs, for instance the Boltzmann equation for dilute gases, the Landau or the Balescu–Guernsey–Lenard equations in plasma physics, or the wave kinetic equation for weak turbulence theory. A few works go beyond the average evolution and describe, for instance, Gaussian fluctuations. However, for many physical systems, rare events can be of importance, and Gaussian fluctuations are not relevant. This is the case for instance if one wants to understand the irreversibility paradox associated to the kinetic equations, or to understand the dynamics that leads to rare events with big impact.

The aim of this presentation is to describe recent results where we derived explicitly the functional that describes the path large deviations for the empirical measure of dilute gases, plasma, systems of particles with long range interactions, and waves with weak interactions. The associated kinetic equations (the average evolution) are then either the Boltzmann, the Landau, the Balescu–Lenard–Guernsey, or the weak turbulence kinetic equations. After making the classic assumptions in theoretical physics textbooks for deriving the kinetic equation, our derivation of the large deviation functional is exact.

These path large deviation principles give a very nice and transparent new interpretation of the classical irreversibility paradox. This new explanation is fully compatible with the classical one, but it gives a deeper insight.

Although this will not be the subject of this talk, I will take five minutes to review our current work to apply rare event algorithms for studying climate extreme events and abrupt transitions.

References:

- For the large deviations associated to the Boltzmann equation (dilute gazes), and a general introduction (published in J. Stat. Phys. in 2020): F. Bouchet, 2020, Is the Boltzmann equation reversible? A large deviation perspective on the irreversibility paradox and the Boltzmann equation, Journal of Statistical Physics, 181, 515-550
- For the large deviations associated to the Landau equation (plasma below the Debye length): O. Feliachi and F. Bouchet, 2021, Dynamical large deviations for plasma

below the Debye length and the Landau equation, Journal of Statistical Physics, 183, 42.

- For the large deviations associated with the Balescu–Guernsey–Lenard equation (plasma and systems with long range interactions): O. Feliachi and F. Bouchet, 2022, Dynamical Large Deviations for Homogeneous Systems with Long Range Interactions and the Balescu–Guernsey–Lenard Equation, Journal of Statistical Physics 186, 22

- For the large deviations associated with the weak turbulence kinetic equation that describe weakly interacting waves: J. Guioth, F. Bouchet and G. L. Eyink, 2022, Path large deviations for the kinetic theory of weak turbulence, J Stat Phys, 189, 20

- For the large deviations associated with the weak turbulence kinetic equation that describe weakly interacting waves in heterogeneous versions: Y. Onuki, J. Guioth, and F. Bouchet, 2023, Dynamical large deviations for an inhomogeneous wave kinetic theory: linear wave scattering by a random medium, submitted to Ann. Henry. Poincaré, arXiv:2301.03257.