Groupoidal realizability: the topological BHK interpretation of intensional type theory

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We introduce groupoidal realizability. The idea is to equip the Hofmann-Streicher groupoid model with a notion of realizability analogous to how traditional categories of assemblies equip the set model with a notion of realizability. Realizers derive from a "realizer category", which is assumed to have increasingly more structure as we seek to model a more expressive type theory. The key piece of structure is an invertible interval qua internal co-groupoid, which facilitates an abstract fundamental groupoid construction. Objects of a groupoid are realized by points in the fundamental groupoid of some realizing space, isomorphisms in the groupoid are realized by paths in the fundamental groupoid. The category of spaces and homotopy classes of maps is an example of a realizer category. We thus claim that groupoidal realizability formalizes the "topological BHK interpretation" of intensional type theory: types as spaces, terms as points, identity proofs as paths and higher identity proofs as homotopies. Another example of a realizer category comes from the computational setting of game semantics. Finally, we investigate the relationship between "modest groupoids" and "generalized congruences", and thus impredicative and univalent universes.