

Filtered colimits and free groups on sets

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An open problem in HoTT asks to show that the free (higher) group on a set has a set as carrier. While I don't know how to solve this problem, I will tell you how to show that the n -truncation of the carrier is a set for any external n . (Alternatively, one may assume certain infinitary axioms which hold in the simplicial set model.) For this, we develop the theory of filtered colimits synthetically in a (higher) topos. We base this on a more general result: pullbacks form a sound doctrine in the sense of Adamek et al. This means that D -shaped colimits commute with pullbacks exactly if the (higher) category of cocones of any span in D has terminal realization. This property fails for 1-toposes.