MLL multiplicative

$$A, B ::= X \mid A^{\perp} \mid \underset{\text{one}}{1} \mid \underset{\text{tensor}}{A \otimes B} \mid \underset{\text{bottom}}{\perp} \mid \underset{\text{par}}{A \parr B}$$

$$A \multimap B := A^{\perp} \parr B$$

$$\underbrace{A_1, \dots, A_n}_{\text{hypothesis}} \vdash \underbrace{B_1, \dots, B_m}_{\text{thesis}}$$

$$\frac{}{A \vdash A}\,\text{ax} \qquad \frac{\Gamma \vdash A, \Delta \qquad \Sigma, A \vdash \Pi}{\Gamma, \Sigma \vdash \Delta, \Pi}\,\text{cut}$$

$$\frac{\Gamma \vdash A, \Delta}{\Gamma, A^{\perp} \vdash \Delta}\,\perp l \qquad \frac{\Gamma, A \vdash \Delta}{\Gamma \vdash A^{\perp}, \Delta}\,\perp r$$

MLL multiplicative

$$A, B ::= X \mid X^\perp \mid 1 \mid A \otimes B \mid \bot \mid A \parr B$$

1 one | $A \otimes B$ tensor

$\bot$ bottom | $A \parr B$ par

$$1^\perp = \bot$$
$$\bot^\perp = 1$$
$$(A \otimes B)^\perp = A^\perp \parr B^\perp$$
$$(A \parr B)^\perp = A^\perp \otimes B^\perp$$
$$(X)^\perp = X^\perp$$
$$(X^\perp)^\perp = X$$

$$A \multimap B := A^\perp \parr B$$

$$\underbrace{A_1, \dots, A_n}_{\text{hypothesis}} \vdash \underbrace{B_1, \dots, B_m}_{\text{thesis}} \rightsquigarrow \vdash A_1^\perp, \dots, A_n^\perp, B_1, \dots, B_m$$

$$\dfrac{}{\vdash A^\perp, A}\,ax \qquad \dfrac{\vdash A, \Delta \qquad \vdash A^\perp, \Pi}{\vdash \Delta, \Pi}\,cut$$

$$\dfrac{}{\vdash 1}\,1r \qquad \dfrac{\vdash A, \Gamma \qquad \vdash B, \Delta}{\vdash A \otimes B, \Gamma, \Delta}\,\otimes$$

$$\dfrac{\vdash \Gamma}{\vdash \bot, \Gamma}\,\bot \qquad \dfrac{\vdash A, B, \Gamma}{\vdash A \parr B, \Gamma}\,\parr$$

$$\dfrac{\vdash \Gamma, A, B, \Delta}{\vdash \Gamma, B, A, \Delta}\,ex$$

additives (connectives for choices) (MALL)

$\top$ top	$A \,\&\, B$ with
0 zero	$A \oplus B$ plus

$$\top^\perp = 0 \qquad (A \,\&\, B)^\perp = A^\perp \oplus B^\perp$$
$$0^\perp = \top \qquad (A \oplus B)^\perp = A^\perp \,\&\, B^\perp$$

$$\frac{}{\vdash \top, \Gamma}\ \text{top} \qquad \frac{\vdash A, \Gamma \quad \vdash B, \Gamma}{\vdash A \,\&\, B, \Gamma}\ \&$$

zero no right intro rule

$$\frac{\vdash \Gamma, A}{\vdash \Gamma, A \oplus B}\ \oplus 1 \qquad \frac{\vdash \Gamma, B}{\vdash \Gamma, A \oplus B}\ \oplus 2$$

$$\otimes \longrightarrow \parr$$

$$\cfrac{\cfrac{\cfrac{\vdash A, \Gamma \qquad \vdash B, \Gamma}{\vdash A \otimes B, \Gamma, \Gamma}\,\otimes}{\quad}\,c}{\vdash A \otimes B, \Gamma}$$

$$\cancel{\cfrac{\vdash \Gamma}{\vdash \Gamma, A}\,w} \qquad \cancel{\cfrac{\vdash \Gamma, A, A}{\vdash \Gamma, A}\,c}$$

$$\parr \xrightarrow{w} \otimes$$

$$\cfrac{\cfrac{\vdash A, \Gamma}{\vdash A, \Gamma, \Delta}\,w \qquad \cfrac{\vdash B, \Delta}{\vdash B, \Gamma, \Delta}\,w}{\vdash A \parr B, \Gamma, \Delta}\,\parr$$

Exponentials: bridge between (M/A)LL

$!A$	$?A$
of-course, bang	why-not

$$(!A)^\perp = ?A^\perp$$

$$(?A)^\perp = !A^\perp$$

$$\frac{\vdash ?\Gamma, A}{\vdash ?\Gamma, !A}\ !\text{promotion}$$

$$\frac{\vdash \Gamma, A}{\vdash \Gamma, ?A}\ ?\text{dereliction}$$

$$\frac{\vdash \Gamma}{\vdash \Gamma, ?A}\ w \qquad \frac{\vdash \Gamma, ?A, ?A}{\vdash \Gamma, ?A}\ c$$

$$!(A \& B) \dashv\vdash \; !A \otimes !B$$

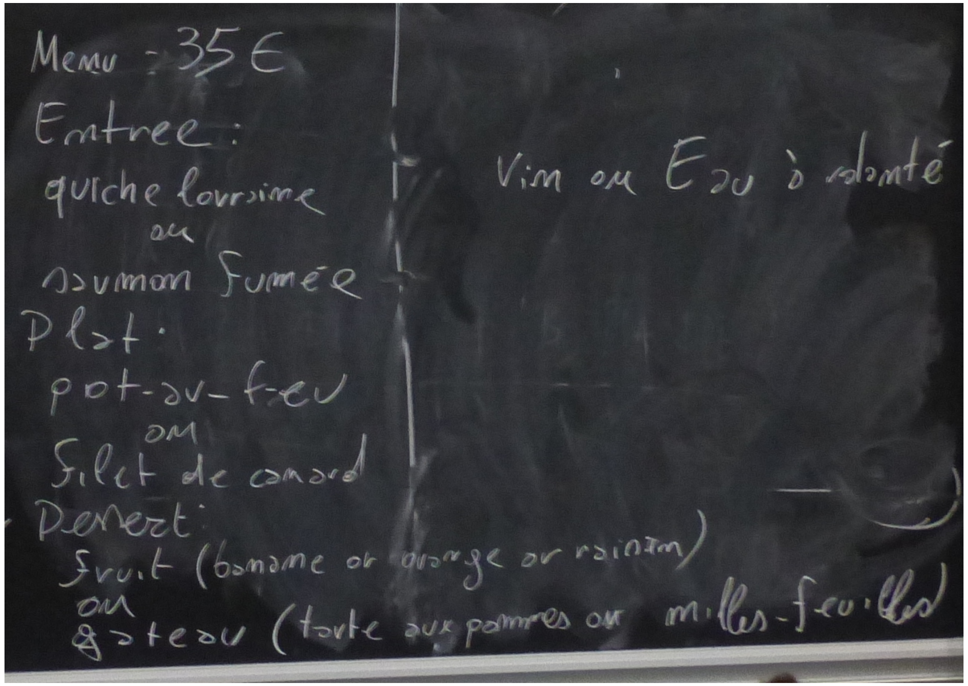
$$?(A \oplus B) \dashv\vdash \; ?A \parr ?B$$

$$!\top \dashv\vdash 1$$

$$?0 \dashv\vdash \bot$$

$$
\dfrac{
  \dfrac{
    \dfrac{
      \dfrac{
        \dfrac{}{\vdash A, A^\perp}\,\text{ax}
      }{\vdash A, A^\perp \oplus B^\perp}\,\oplus 1
    }{\vdash A, ?(A^\perp \oplus B^\perp)}\,?d
  }{\vdash !A, ?(A^\perp \oplus B^\perp)}\,!
  \qquad
  \begin{array}{c} \vdots \;\oplus 2 \\ \vdash !B, ?(A^\perp \oplus B^\perp) \end{array}
}{\vdash ?(A^\perp \oplus B^\perp), ?(A^\perp \oplus B^\perp), !A \otimes !B}\,\otimes
$$

$$
\dfrac{\vdash ?(A^\perp \oplus B^\perp), ?(A^\perp \oplus B^\perp), !A \otimes !B}{\vdash ?(A^\perp \oplus B^\perp), !A \otimes !B}\,?\text{cont}
$$


Menu : 35 €
Entree :
quiche lorraine
ou
saumon fumée
Plat :
pot-au-feu
ou
filet de canard
Dessert :
fruit (banane ou orange ou raisin)
ou
gateau (tarte aux pommes ou milles-feuilles)
Vin ou Eau à volonté

$$(35€)^{\perp} \parr (Q \parr S) \otimes$$
$$\otimes (P \parr F)$$
$$\otimes ((B \oplus O_n \oplus R)$$
$$\parr (T \parr M))$$