

O-minimal preparation theorem

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Preparation theorem for subanalytic functions was introduced in the middle of the 90's of the last century for the construction of Lipschitz stratification. Independently, it appeared in J.-M. Lion and J.-P. Rolin's proof of the elimination of quantifiers for sets defined by logarithmic-exponential, subanalytic, and power functions. It can be understood as a rectilinearization of functions of many variables with respect to a fixed single distinguished variable. This variable can be chosen arbitrarily. The theorem says that any such function can be written on every stratum of a finite subanalytic stratification as a product $f(x, y) = a(x)|y - \theta(x)|^r \text{unit}(x, y)$, where y is the distinguished variable and x stands for the remaining variables. This theorem has been extended to arbitrary polynomially bounded o-minimal structures by L. van den Dries and P. Speissegger.

We divide this mini-course into three parts: motivation, proof, and applications, focusing mainly on the geometric aspects of the theory.