O-minimal expansions of the real field : some basics.

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1. Definition and basic properties

In the fist session, we introduce the definition of an o-minimal expansion of the field of real, and the related basic tools: finiteness properties, selection lemma, C^k cell decomposition. Main references: [1, 2].

2. Polynomial boundedness versus exponential.

In this session, we discuss the dichotomy growth theorem of Chris Miller: an o-minimal structure is either polynomially bounded or defines the exponential function. We introduce the Hardy field of an o-minimal structure, and investigate the assympotic of definable functions. We deduce a version of Łojasciwicz inequality in polynomially bounded structures. Main references: [3, 4].

3. Quasi-analytic classes and o-minimality.

In this session we deal with the link between quasi-analytic classes and o-minimality. We review some examples of o-minimal structures generated by quasi-analytic classes. Main reference: [5].

References

[1] Michel Coste, An introduction to o-minimal geometry, Instituti editoriali e poligrafici inter- nazionali, 2000.

[2] van den Dries, Lou Tame topology and o-minimal structures. London Mathematical Society Lecture Note Series, 248. Cambridge University Press, Cambridge

[3] Chris Miller, Exponentiation is hard to avoid. Proc. Amer. Math. Soc. 122 (1994), no. 1, 257–259.

[4] Chris Miller, Basics of o-minimality and Hardy fields. Lecture notes on o-minimal structures and real analytic geometry, 43–69, Fields Inst. Commun., 62, Springer, New York, 2012.

[5] Rolin J.-P., Speissegger P., Wilkie, A. J., Quasianalytic Denjoy-Carleman classes and o-minimality, J. Amer. Math. Soc. 16 (2003), no. 4, 751–777.