On C^r parameterizations in and outside o-minimality, and point counting

Raf Cluckers,

Université de Lille

C^r parameterizations were discovered by Yomdin and Gromov in the real semi-algebraic case in the late eighties (with several alternative and more detailed proofs to follow, e.g. by Burguet, Pila-Wilkie, Kocel-Pawłucki-Valette, Binyamini-Novikov). Such parameterizations were initially used in the study of entropy by Yomdin and Gromov. More recently, their generalizations to the o-minimal setting by Pila and Wilkie have given striking diophantine applications. Since then, many variants of the C^r parameterizations and their applications have emerged, both in o-minimal and other settings. I will discuss some of such recent variants. One such variant is an improved result on parameterizations in the subanalytic case, by Pila, Wilkie and myself. This came in fact in analogy to results on the subanalytic sets in the p-adics, by Comte, Forey, Loeser and myself, leading to improved point counting results on subanalytic sets (both real and p-adic), in a direction towards Wilkie's Conjecture (namely with polylogarithmic instead of subexponential bounds). In fact, point counting on definable sets in non-archimedean settings has many faces. (All mimicking some aspects of point counting in o-minimal settings.) For sets living in Q_p^n, one can count actual rational points of bounded height, but for sets in $C((t))^n$, one rather "counts" the polynomials in t of bounded degree. But, the latter can be of infinite cardinality! For C((t)), we discuss: 1) the setting of subanalytic sets, where we show finiteness of point counting (on transcendental parts) but where growth can be aribitrarily fast with the degree in t; 2) the setting of Pfaffian sets, which is new in the non-archimedean world and for which we show an analogue of Wilkie's Conjecture in all dimensions; 3) the (axiomatic) Hensel minimal setting, which is most general and where finiteness starts to fail, even for definable transcendental curves. In this infinite case, one bounds the dimension rather than the (infinite) cardinality. Items 1 – 3 represent joint work with Binyamini, Novikov, with Halupczok, Rideau, Vermeulen, and separate work by Cantoral-Farfan, Nguyen, Vermeulen.