

*Jean-Morlet Chair - Conference*  
Arithmetic Statistics - Statistiques arithmétiques

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Title: *Murmurations of L-functions*

Abstract: While conducting a series of number-theoretic machine learning experiments, He, Lee, Oliver, and Pozdnyakov noticed a curious oscillation in the averages of Frobenius traces of elliptic curves over  $\mathbb{Q}$ . If one computes the average value of  $a_p(E)$  for  $E/\mathbb{Q}$  of fixed rank with conductor in a given range, as  $p$  increases the average of  $a_p(E)$  oscillates with a decaying frequency determined by the conductor. The shape of these oscillations depends on the parity and magnitude of the rank. That the rank influences the distribution of Frobenius traces has long been known (indeed, this was the impetus for the numerical experiments of Birch and Swinnerton-Dyer), but these oscillations do not appear to have been noticed previously. This is perhaps due to the critical role played by the conductor: if one uses height bounds rather than conductor bounds the oscillations are no longer visible.

I will present preliminary results from an ongoing investigation of this phenomenon, which is remarkably robust and not specific to elliptic curves. One finds similar oscillations in the averages of Dirichlet coefficients for many types of arithmetic L-functions when organized by conductor and root number, including those associated to modular forms (not necessarily of weight 2) and abelian varieties (not necessarily of dimension 1).