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Title: *Vanishing of twisted L-functions of elliptic curves over function fields*

Abstract: Let E be an elliptic curve over the rationals, and let χ be a Dirichlet character of order ℓ for some odd prime ℓ. Heuristics based on the distribution of modular symbols and random matrix theory have led to conjectures predicting that the vanishing of the twisted L-functions $L(E,\chi,s)$ at $s=1$ is a very rare event (David-Fearnley-Kisilevsky and Mazur-Rubin). In particular, it is conjectured that there are only finitely many characters of order $\ell>5$ such that $L(E,\chi,1)=0$ for a fixed curve E. We investigate the case of elliptic curves over function fields. For Dirichlet L-functions over function fields, Li and Donepudi-Li have shown how to use the geometry to produce infinitely many characters of order $l\geq 2$ such that the Dirichlet L-function $L(\chi,s)$ vanishes at $s=1/2$, contradicting (the function field analogue of) Chowla's conjecture. We show that their work can be generalized to constant curves $E/\mathbb{F}_q(t)$, and we show that if there is one Dirichlet character χ of order ℓ such that $L(E,\chi,1)=0$, then there are infinitely many, leading to some specific examples contradicting (the function field analogue of) the number field conjectures on the vanishing of twisted L-functions. Such a dichotomy does not seem to exist for general curves over $\mathbb{F}_q(t)$, and we produce empirical evidence which suggests that the conjectures over number fields also hold over function fields for non-constant $E/\mathbb{F}_q(t)$. This is joint work with A. Comeau-Lapointe, C. David, and W. Li.