Aleksandra Kuznetsova

Regularizations of birational automorphisms

We consider a birational automorphism f of an algebraic variety X and ask a natural question: is there a birational model of X such that f induces a regular automorphism on it? Weil proved that if the growth of the sequence of degree of f^n is bounded when n tends to infinity, then f is regularizable. On the contrary, if the growth of degrees of f is exponential it is hard to separate regularizable and non-regularizable automorphisms. There is a beautiful theorem by Diller and Favre: when X is a surface and degrees of f grow exponentially, there is a model of X and a nef divisor D in X which is an eigenvalue for the action of f^* on NS(X), and f is regularizable iff D^2 = 0..

We study the case when X is a threefold and f is a pseudo-automorphism; thus, degrees of f^n grow with an exponent which equals the greatest eigenvalue of the action of f^* on NS(X). In this case the divisor D which is the corresponding eigenvector can be not nef even in case when f is regularizable. However, we will prove that the orbit of the D-negative curve cannot be infinite. In particular, this allows us to show that the example of pseudo-automorphism constructed by Blanc is non-regularizable.