

# DYNAMICAL SYSTEMS AND THEIR ALGEBRA

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Abstract: Two natural operations to combine finite, discrete dynamical systems (or, from the opposite perspective, to decompose them into smaller ones) are their alternative and their synchronous execution. By choosing these two operations as sum and product, we obtain a commutative semiring structure. The resulting algebra is quite complex; for instance, most systems are irreducible, but those that are reducible sometimes admit multiple factorisations into irreducibles. We present some results related to the resolution of polynomial equations on the semiring of dynamical systems, which turns out to be undecidable in general, and NP-complete even for linear equations. Furthermore, we describe some recent research on the existence of prime dynamical systems; these are systems that, whenever they appear in the decomposition of a system, they necessarily appear in all the others as well, and are thus fundamental building blocks. We do not know yet if prime systems exist, or even if there exists a computable primality test, but several interesting classes of systems have been proved to be non-prime, including systems only consisting of limit cycles (which includes the asymptotic behaviour of any dynamical system).