

ISOMETRIES OF THE HYPERCUBE

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Abstract: Given S a Boolean finite dynamical system (FDS), and f an isometry of the hypercube, we consider the conjugated FDS $\Phi = f \circ S \circ \text{inv}(f)$. This conjugate conserves all the dynamical properties of S . Moreover, the regulatory graphs of two conjugated FDS are similar: they have the same topology; edges may switch their signs, but signs of circuits remain unchanged. The logical rules may be modified; for example, logical operators OR and AND may be interchanged between two conjugated FDS, but not OR and XOR. The group of all the isometries of the hypercube then defines classes of Boolean FDS, gathering all the conjugates of a given Boolean FDS, in other words gathering all the isometric dynamics. Thus, we classify the set of Boolean FDS on the basis of those isometries. We can then restrict the dynamical analysis of all the Boolean FDS to one representative per class, and thereby considerably restrict the dynamical analysis of all the Boolean FDS. Relying on invariants properties, we propose a constructive method to provide, given a FDS, a representative regulatory graph of its class of FDS under isometries, that fits the addressed questions. We illustrate the efficiency of the method in concrete situations. For instance, the analysis of well-known motifs is strongly improved thanks to the reduction of the space to explore.