ALGEBRAIC STRUCTURES HIDING IN AUTOMATA NETWORKS

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Abstract: I will begin this talk with a problem from dynamical algebraic combinatorics about toggling independent sets, and frame it as a finite asynchronous cellular automata using ECA rule 1. In analyzing the dynamics, we find two local invariants that define bijections of the ?live entries? in the orbits. This defines a simply transitive action of an infinite abelian group. In other words, it endows the live entries with the structure of a Cayley diagram. By studying this action, and using the theory of covering spaces from algebraic topology (which I will not assume prior knowledge of), we are able to classify many combinatorial properties and relationships about the structure of the orbits. Happily, this is all a special case of a more general theory, applicable to other automata networks and combinatorial dynamical systems, which seems to be mostly unexplored. I will pose a number of interesting avenues for future research, in hopes to entice members of the audience. Though it will not be necessary to follow this talk, I or (most likely) a colleague will give a survey talk at AUTOMATA about the connections between dynamical algebraic combinatorics and cellular automata, which will give this problem more context and motivation. Both talks will be filled with lots of examples, colorful pretty pictures, and puns.