Numerical solutions for image processing problems Part II

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Level set equation [S], [OS]

- level set equation: $u_t - |\nabla u| \nabla \cdot \left(\frac{\nabla u}{|\nabla u|} \right) = 0$
- unknown function $u(t, x)$ is defined in the domain $Q_T = I \times \Omega$
- we consider Neumann (or Dirichlet) boundary conditions $\partial_\nu u = 0$ on $I \times \partial\Omega$ $(u(t, x) = 0$ on $I \times \partial\Omega)$
- and the initial condition $u(0, x) = u^0(x)$
- hypothesis (H):
  - Ω is a rectangular domain in $\mathbb{R}^d, d = 2, 3$
  - $u^0 \in H_0^1(\Omega)$

Time approximation [HMS],[EHM]

- we set the uniform time step $\tau = \frac{T}{N_T}$

denote u^n as an approximation of $u(t,x)$ at time $t_n = n\tau$

- first time derivative is replaced by the backward difference $\frac{u^n - u^{n-1}}{\tau}$
- level set equation can be rewritten into the form of **semi-implicit scheme:** $\frac{1}{|\nabla u^{n-1}|}\frac{u^n - u^{n-1}}{\tau} = \nabla \cdot \left(\frac{\nabla u^n}{|\nabla u^{n-1}|}\right)$

Space approximation

finite volume method - approximated solution is piecewise constant function in space and time [EGH],[EHM]

We can denote p as the finite volume with measure of $m(p)$, e^{pq} as the edge (face) between two finite volumes p and q and $N(p)$ as the set of all finite volume neighbors

- by application of the **divergence theorem we get the integral formulation**
$$\int_p \frac{1}{|\nabla u^{n-1}|} \frac{u^n - u^{n-1}}{\tau} dz = \sum_{q \in N(p)} \int_{e^{pq}} \frac{1}{|\nabla u^{n-1}|} \frac{\partial u^n}{\partial \nu} ds$$

Regularized level set equation

$$u_t(t,x) - f(|\nabla u(t,x)|)\nabla\left(\frac{\nabla u(t,x)}{f(|\nabla u(t,x)|)}\right) = 0 \;\; t \in I, \;\; x \in \Omega,$$

Evans-Spruck regularization [ES] $|\nabla u|_\varepsilon = \sqrt{\varepsilon^2 + |\nabla u|^2}$.

For convergence study Eymard regularization assumption [EHS]

$$f(z) = \min(\sqrt{z^2+\varepsilon^2}, b),$$

with fixed regularization parameter $\varepsilon > 0$ and another real fixed parameter b, $\varepsilon < b$.

DDS in 2D Andreianov B., Bendahmare M., Karlsen K. H., Boyer F., Hubert F.:[ABK], [ABH]

Discrete Duality Finite Volume Scheme (DDS)

We consider two meshes

- **original mesh** - we denote V_{ij} as the finite volume with measure of $m(V_{ij}) = h^2$, e_{ij}^{pq}, $m(e_{ij}^{pq}) = h$ as the edge between two finite volumes ij and $i+p, j+q$ $p, q = 0, 1, -1$ and $|p| + |q| = 1$;
  numerical solution on finite volume V_{ij} at time t^n is denoted as u_{ij}^n
- **dual mesh** - we denote $\overline{V}_{ij}$ as the finite volume with measure of $m(\overline{V}_{ij}) = h^2$, $\overline{e}_{ij}^{pq}$, $m(\overline{e}_{ij}^{pq}) = h$ as the edge between two finite volumes p and q;
  numerical solution on finite volume $\overline{V}_{ij}$ at time t^n is denoted as $\overline{u}_{ij}^n$

Original and dual mesh in DDS

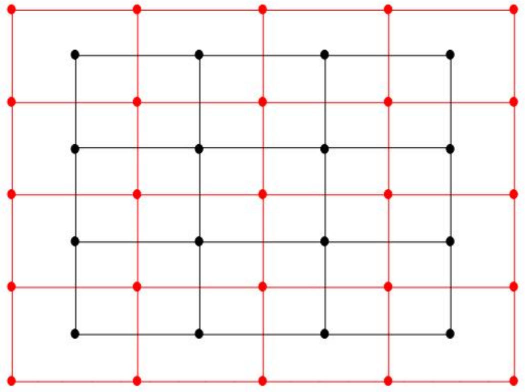


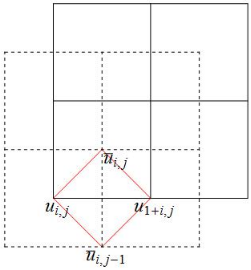
Figure: Original (red rectangles) and dual (black rectangles) mesh

DDS for level set equation in 2D Kotorová H [K1], [HK1]

$$\int_{V_{ij}} \frac{1}{|\nabla u^{n-1}|} \frac{u^n - u^{n-1}}{\tau} = \sum_{|p|+|q|=1} \int_{e_{ij}^{pq}} \frac{1}{|\nabla u^{n-1}|} \frac{\partial u^n}{\partial \nu} ds$$

$$\int_{\overline{V}_{ij}} \frac{1}{|\nabla \overline{u}^{n-1}|} \frac{\overline{u}^n - \overline{u}^{n-1}}{\tau} = \sum_{|p|+|q|=1} \int_{\overline{e}_{ij}^{pq}} \frac{1}{|\nabla \overline{u}^{n-1}|} \frac{\partial \overline{u}^n}{\partial \nu} ds$$

Gradient approximation in DDS [ABK]



- $|\nabla u_{ij}^{n-1}|^2 = \left(\frac{u_{i+1,j} - u_{ij}}{h}\right)^2 + \left(\frac{\overline{u}_{ij} - \overline{u}_{i,j-1}}{h}\right)^2$
- $Q_{ij}^{pq;n-1} = \sqrt{\varepsilon^2 + |\nabla u_{ij}^{n-1}|^2}$
- $AQ_{ij} = \frac{1}{4} \sum\limits_{|p|+|q|=1} Q_{ij}^{pq;n-1}$
- $\int\limits_{V_{ij}} \frac{1}{|\nabla u^{n-1}|} \frac{u_{ij}^n - u_{ij}^{n-1}}{\tau} dx \approx \frac{h^2}{AQ_{ij}} \frac{u_{ij}^n - u_{ij}^{n-1}}{\tau}$
- $\sum\limits_{|p|+|q|=1} \int\limits_{e_{ij}^{pq}} \frac{1}{|\nabla u^{n-1}|} \frac{\partial u^n}{\partial \nu} ds \approx$

  $\sum\limits_{|p|+|q|=1} h \frac{1}{Q_{ij}^{pq;n-1}} \frac{u_{i+p,j+q}^n - u_{ij}^n}{h}$

similarly also for the dual mesh

Linear system of equations in DDS Kotorová D. , H. (2011), (2013) [K1][HK]

initial values

$$u_{ij}^0 = \frac{1}{m(V_{ij})} \int\limits_{V_{ij}} u^0(x)\,\mathrm{d}x \quad \forall V_{ij} \in \mathcal{T}_h, \tag{1}$$

$$\overline{u}_{ij}^0 = \frac{1}{m(\overline{V}_{ij})} \int\limits_{\overline{V}_{ij}} u^0(x)\,\mathrm{d}x \quad \forall \overline{V}_{ij} \in \overline{\mathcal{T}_h}.$$

$$\frac{u_{ij}^n \; m(V_{ij})}{AQ_{ij}^{n-1}} + \tau \sum_{|p|+|q|=1} \frac{(u_{ij}^n - u_{i+p,j+q}^n)\; m(e_{ij}^{pq})}{Q_{ij}^{pq;n-1}\; h} = \frac{m(V_{ij})\; u_{ij}^{n-1}}{AQ_{ij}^{n-1}}$$

$$\frac{\overline{u}_{ij}^n \; m(\overline{V}_{ij})}{\overline{AQ}_{ij}^{n-1}} + \tau \sum_{|p|+|q|=1} \frac{(\overline{u}_{ij}^n - \overline{u}_{i+p,j+q}^n)\; m(\overline{e}_{ij}^{pq})}{\overline{Q}_{ij}^{pq;n-1}\; \overline{h}} = \frac{m(\overline{V}_{ij})\; \overline{u}_{ij}^{n-1}}{\overline{AQ}_{ij}^{n-1}}$$

Numerical Solution

The function $u_{\tau,h}$ is in time and space piecewise constant function defined as follows:

$$u_{\tau,h}(t,x) = \frac{1}{2}\left(u_{\tau,h,V}(t,x) + u_{\tau,h,\overline{V}}(t,x)\right),$$

where

$$u_{\tau,h,V}(t,x) = u_{ij}^n \text{ for } x \in V_{ij},\ t \in ((n-1)\tau, n\tau],$$

$$u_{\tau,h,\overline{V}}(t,x) = \overline{u}_{ij}^n \text{ for } x \in \overline{V}_{ij},\ t \in ((n-1)\tau, n\tau].$$

$$\nabla u_{\tau,h}(t,x) = \begin{cases} \nabla u_{ij}^n \text{ for } x \in D_{ij},\ t \in ((n-1)\tau, n\tau] \\ \nabla \overline{u}_{ij}^n \text{ for } x \in \overline{D}_{ij},\ t \in ((n-1)\tau, n\tau] \end{cases}$$

Stability 1 Kotorová D. , H. (2013) [HK]

Lemma

Let the hypotheses (H) *hold. Then there exist unique solutions*

$$u_h^n = (u_{11}^n, \dots, u_{N_1N_2}^n), \quad \overline{u}_h^n = (\overline{u}_{00}^n, \dots, \overline{u}_{N_1N_2}^n)$$

of the proposed scheme for any value of the regularization parameter $\varepsilon > 0$ *and for any time step* $n = 1, \dots, N_T$. *Moreover, for the fully discrete numerical solution* $u_{\tau,h}$ *the following estimates hold*

$$\|u_{\tau,h}\|_{L_\infty(I\times\Omega)} \le \|u_h^0\|_{L_\infty(\overline{\Omega})}.$$

Stability 2 [HK]

Lemma

Let the hypotheses (H) *hold. Then for the solution of proposed scheme the following stability results hold for* $m = 1, 2, \ldots, N$*:*

$$\sum_{n=1}^{m} \left(\sum_{V_{ij} \in \mathcal{T}} \frac{\left(u_{ij}^{n} - u_{ij}^{n-1}\right)^2}{AQ_{ij}^{n-1}} \frac{h^2}{\tau} + \sum_{\overline{V}_{ij} \in \overline{\mathcal{T}}} \frac{\left(\overline{u}_{ij}^{n} - \overline{u}_{ij}^{n-1}\right)^2}{\overline{AQ}_{ij}^{n-1}} \frac{h^2}{\tau} \right) +$$

$$\sum_{D_{ij} \in \mathcal{D}_h} \left|\nabla u_{ij}^{m}\right| h^2 + \sum_{\overline{D}_{ij} \in \overline{\mathcal{D}}_h} \left|\nabla \overline{u}_{ij}^{m}\right| h^2 \leq C$$

where C *is a generic constant and depends only on data of the problem, not on* h *or* τ*.*

Convergence Kotorová D. , H. (2013)[HK]

Theorem

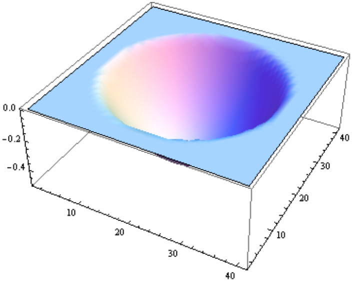
Let hypothesis (H) *be fulfilled. Then there exists a subsequence of* $(\tau_m, h_m)_{m\in\mathbf{N}}$, *again denoted by* $(\tau_m, h_m)_{m\in\mathbf{N}}$ *and there exists a function* $\overline{u} \in L^\infty((0,T); H_0^1(\Omega)) \cap C^0((0,T); L^2(\Omega))$, *such that*

$$\begin{aligned} \overline{u}_t &\in L^2((0,T)\times\Omega), \\ u(0,.) &= u^0, \\ u_{\tau_m,h_m} \longrightarrow \overline{u} &\in L^\infty((0,T); L_2(\Omega)), \\ u_{\tau_m,h_m} \rightharpoonup \overline{u} &\in L^\infty((0,T); H_0^1(\Omega)) \end{aligned}$$

Cut paraboloid - Obermann solution

- cut paraboloid with zero Neumann boundary conditions
- numerical solution on the square mesh $n \times n$
- exact solution given by $\min\left\{\frac{1}{2}(x^2+y^2+1)-t, 0\right\}$

Cut paraboloid - Obermann solution



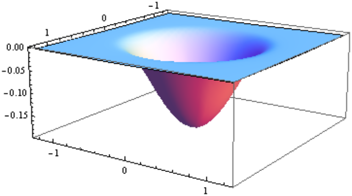


Figure: Initial condition and solution in time $t = 0.3125$

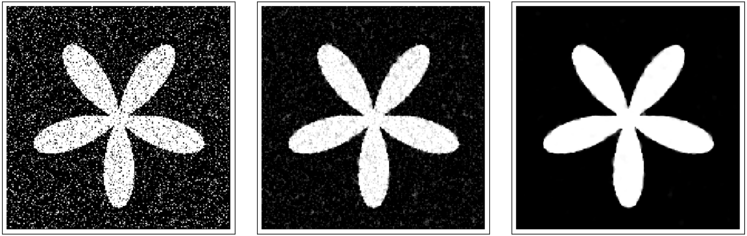
Cut paraboloid - Obermann solution

n	L_2 error	EOC L_2 error	L_∞ error	EOC L_∞ error
10	$6.755e^{-2}$	–	$1.618e^{-1}$	–
20	$3.401e^{-2}$	0.9899	$8.507e^{-2}$	0.9275
40	$1.717e^{-2}$	0.9861	$4.218e^{-2}$	1.0121
80	$8.730e^{-3}$	0.9758	$2.108e^{-2}$	1.0007
160	$4.421e^{-3}$	0.9816	$1.057e^{-2}$	0.9959
320	$2.229e^{-3}$	0.9879	$5.298e^{-3}$	0.9965

Cut paraboloid - Obermann solution

n	L_2 gradient error	EOC L_2 gradient error	L_∞ gradient error	EOC L_∞ gradient error
10	$2.492e^{-1}$	–	$5.000e^{-1}$	–
20	$2.129e^{-1}$	0.2271	$4.339e^{-1}$	0.2046
40	$1.748e^{-1}$	0.2845	$3.506e^{-1}$	0.3075
80	$1.409e^{-1}$	0.3110	$2.757e^{-1}$	0.3467
160	$1.126e^{-1}$	0.3235	$2.188e^{-1}$	0.3335
320	$8.973e^{-2}$	0.3275	$1.738e^{-1}$	0.3322

Cinquefoil with 20 percent salt and pepper noise



Barbara

Original and dual mesh in DDS in the "Hermeline" scheme

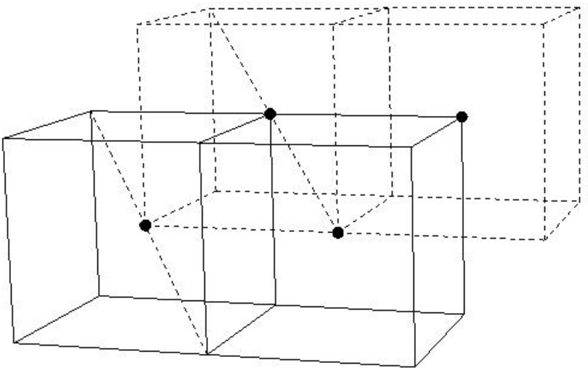
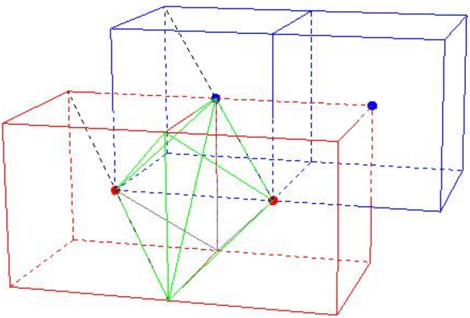


Figure: Original (solid lines cubes) and dual (dashed lines cubes) mesh

Gradient approximation in DDS in the "Hermeline" scheme



B. Andreianov, F. Hermeline

- $$\int\limits_{V_{ijk}} \frac{1}{|\nabla u^{n-1}|} \frac{u^n_{ijk} - u^{n-1}_{ijk}}{\tau} dx \approx \frac{h^3}{AQ_{ijk}} \frac{u^n_{ijk} - u^{n-1}_{ijk}}{\tau}$$
- $$AQ_{ijk} = \frac{1}{6} \sum_{|p|+|q|+|r|=1} Q^{pqr;n-1}_{ijk}$$
- $$Q^{pqr;n-1}_{ijk} = \sqrt{\varepsilon^2 + |\nabla u^{n-1}_{ijk}|^2}$$

Gradient approximation in DDS in the "Hermeline" scheme

- $|\nabla u_{ijk}^{n-1}|^2 =$
$$\left(\frac{u_{i+1,j,k}^{n-1} - u_{ijk}^{n-1}}{h}\right)^2 + \left(\frac{v_{ijk} + v_{i,j,k-1} - v_{i,j-1,k} - v_{i,j-1,k-1}}{2h}\right)^2 + \left(\frac{v_{i,j-1,k} + v_{ijk} - v_{i,j-1,k-1} - v_{i,j,k-1}}{2h}\right)^2$$

- $|\nabla v_{ijk}^{n-1}|^2 = \left(\frac{v_{i+1,j,k}^{n-1} - v_{ijk}^{n-1}}{h}\right)^2 +$
$$\left(\frac{u_{i+1,j+1,k+1} + u_{i+1,j+1,k} - u_{i+1,j,k+1} - u_{i+1,j,k}}{2h}\right)^2 + \left(\frac{u_{i+1,j,k+1} + u_{i+1,j+1,k+1} - u_{i+1,j,k} - u_{i+1,j+1,k}}{2h}\right)^2$$

Original and dual mesh in DDS in the "Coudiére-Hubert" scheme

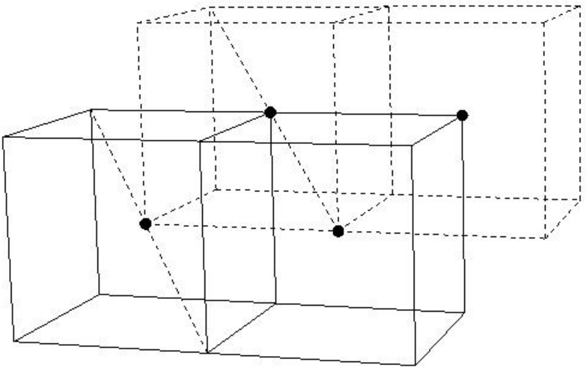


Figure: Original (solid lines cubes) and dual (dashed lines cubes) mesh

Face-edge mesh in DDS in the "Coudiére-Hubert" scheme

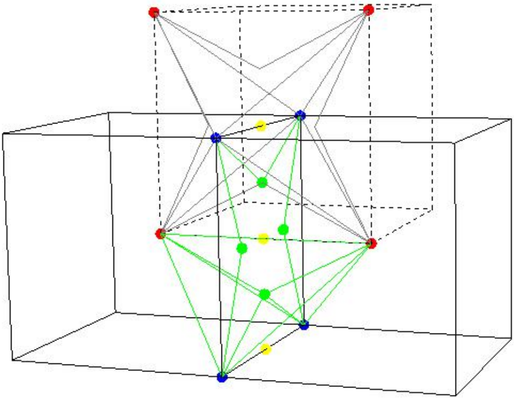
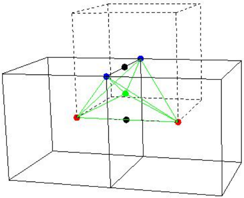


Figure: Face volume (green) and edge volume (gray) in the original mesh

Gradient approximation in DDS in the "Coudiére-Hubert" scheme



- $\int\limits_{V_{ijk}} \frac{1}{|\nabla u^{n-1}|} \frac{u_{ijk}^{n}-u_{ijk}^{n-1}}{\tau} dx \approx \frac{h^3}{AQ_{ijk}} \frac{u_{ijk}^{n}-u_{ijk}^{n-1}}{\tau}$
- $AQ_{ijk} = \frac{1}{6} \sum\limits_{|p|+|q|+|r|=1} Q_{ijk}^{pqr;n-1}$
- $Q_{ijk}^{pqr;n-1} = \sqrt{\varepsilon^2 + |\nabla u_{ijk}^{n-1}|^2}$
- $\nabla D_1 X_{ijk} = \left(\frac{u_{i+1,j,k}^{n}-u_{i,j,k}^{n}}{h}, \frac{v_{i,j,k}^{n}-v_{i,j-1,k}^{n}}{h}, \frac{zy_{i,j,k}^{n}-wx_{i,j,k}^{n}}{\frac{h}{2}} \right)$

Cut paraboloid - Obermann solution

- cut paraboloid with zero Neumann boundary conditions
- numerical solution on the square mesh $n \times n \times n$
- exact solution given by $\min \left\{\frac{1}{2}(x^2 + y^2 + z^2 - 1) - 2t, 0\right\}$

Cut paraboloid - Obermann solution Kotorová (2013)

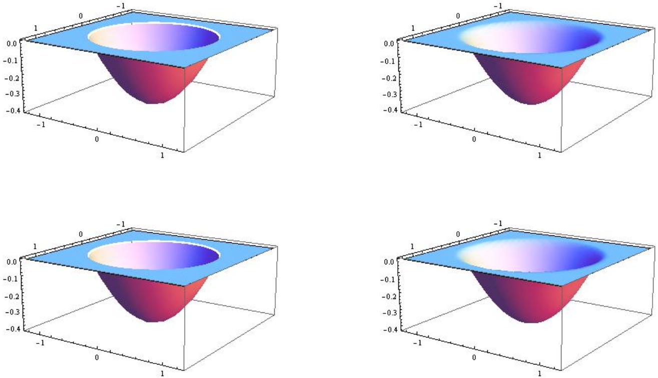


Figure: Exact (left) and numerical (right) solution after 10 time steps obtained by "Hermeline" (top) and "Coudiére-Hubert" (bottom) scheme

Cut paraboloid - Obermann solution

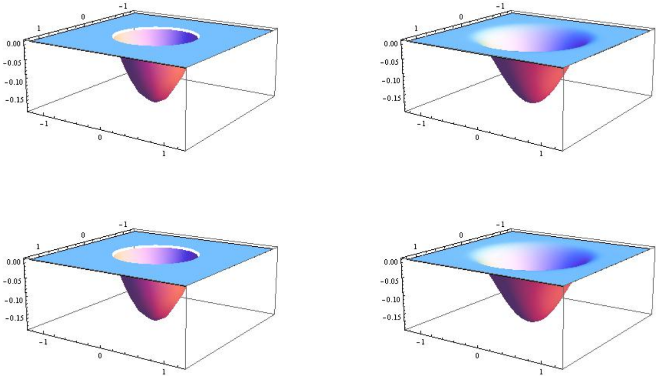


Figure: Exact (left) and numerical (right) solution after 40 time steps obtained by "Hermeline" (top) and "Coudiére-Hubert" (bottom) scheme

Cut paraboloid - Obermann solution

N	L_2 error	EOC L_2 error	L_2 gradient error	EOC L_2 gradient error
10	$6.9985e^{-2}$	–	$2.7369e^{-1}$	–
20	$3.7869e^{-2}$	0.9084	$2.2711e^{-1}$	0.2691
40	$1.9153e^{-2}$	0.9611	$1.7367e^{-1}$	0.3871
80	$9.7404e^{-3}$	0.9755	$1.3319e^{-1}$	0.3829

Table: EOC and errors obtained by "Hermeline" DDS scheme

Cut paraboloid - Obermann solution

N	L_2 error	EOC L_2 error	L_2 gradient error	EOC L_2 gradient error
10	$6.5323e^{-2}$	–	$2.6010e^{-1}$	–
20	$3.4497e^{-2}$	0.9211	$2.1560e^{-1}$	0.2707
40	$1.7830e^{-2}$	0.9522	$1.7224e^{-1}$	0.3239
80	$1.0902e^{-2}$	0.7097	$1.3932e^{-1}$	0.3060

Table: EOC and errors obtained by "Coudiére-Hubert" DDS scheme

AMSS model Carlini Feretti (2013) [CF]

Unknown function $u(t,x)$ defined in the domain $Q_T = Ix\Omega$
AMSS equation: $u_t - |\nabla u| \left(\nabla \cdot \left(\frac{\nabla u}{|\nabla u|}\right)\right)^{\frac{1}{3}} = 0.$
Dirichlet boundary condition $u(t,x) = 0$ on $I \times \partial\Omega$
Initial condition $u(0,x) = u^0(x)$ on Ω
$\Omega \subset \mathbb{R}^d$, with $d \in \mathbf{N}$, and $\partial\Omega$ is its boundary.
Regularized problem is of the form [ES],[EHM]

$$u_t - f(|\nabla u|) \left(\operatorname{div} \left(\frac{\nabla u}{f(|\nabla u|)}\right)\right)^{\frac{1}{3}} = 0, \text{ a.e. } (t,x) \in I \times \Omega,$$

$$f(s) = \min\{\sqrt{s^2 + a}, b\} \quad a > 0, b > 0.$$

Numerical approximation using FVM (2020), (2021) [H], [HM]

For arbitrary finite volume p with the boundary ∂p and a unit outward normal $\mathbf{n_p}$ we obtain

$$\int_p \left(\frac{u_t}{f(|\nabla u|)} \right)^3 dx - \int_{\partial p} \frac{\partial u}{\partial \mathbf{n_p}} \frac{1}{f(|\nabla u|)} ds = 0,$$

On finite volume p and $(n\tau, (n+1)\tau)$ we denote
$\delta_t u_p^n$-approximation of time derivative
$N_p(u^l)$ - approximation of gradient for $l = n$ explicit scheme or semi implicit scheme$l = n + 1$ implicit scheme

$$\left(\delta_t u_p^n\right)^3 \frac{1}{(f(N_p(u^l))^3}|p| - \sum_{\sigma\in\mathcal{E}_p} \frac{(u_q^l - u_p^l)|\sigma|}{f(N_p(u^l))\ d_{p\sigma} + f(N_q(u^l))\ d_{q\sigma}} = 0$$

Second term is in fact the approximation of curvature on finite volume p.

$$K_p^{l,k} = \frac{1}{|p|}\sum_{\sigma\in\mathcal{E}_p} \frac{|\sigma|(u_q^k - u_p^k)}{d_{p\sigma} f(N_p(u^l)) + d_{q\sigma} f(N_q(u^l)}$$

for $k = l = n+1$ fully implicit scheme, for $k = n, l = n+1$ semi-implicit scheme Finally

$$\left(\delta_t u_p^n\right)^3 \frac{1}{(f(N_p(u^l))^3}|p| - K_p^{l,k}|p| = 0,\ \forall p,\ \forall n \in \mathbf{N},$$

Approximation of time derivative:

$$\delta_t u_p^n = f(N_p(u^l)) \left(K_p^{l,k}\right)^{\frac{1}{3}}$$

Now if we approximate

$$\left(\delta_t u_p^n\right)^3 \approx \frac{u_p^{n+1} - u_p^n}{\tau} f(N_p(u^l))^2 (K_p^{l,k})^{\frac{2}{3}}$$

we have

$$\frac{u_p^{n+1} - u_p^n}{\tau} \frac{\left(f(N_p(u^l))(K_p^{l,k})^{\frac{1}{3}}\right)^2}{f(N_p(u^l))^3} |p| - K_p^{l,k}|p| = 0$$

and

$$\frac{u_p^{n+1} - u_p^n}{\tau} |p| - f(N_p(u^l))(K_p^{l,k})^{\frac{1}{3}} |p| = 0.$$

Numerical schemes [HM]

Explicit scheme

$$u_p^{n+1} = u_p^n + \tau f(N_p(u^n)) \left(K_p^{n,n}\right)^{\frac{1}{3}}$$

Fully-implicit scheme

$$u_p^{n+1} - \tau f(N_p(u^{n+1})) \left(K_p^{n+1,n+1}\right)^{\frac{1}{3}} = u_p^n$$

This nonlinear algebraic system we can compute in iterative way [HM]

Numerical schemes [HM]

Semi-implicit scheme

$$u_p^{n+1} = u_p^n + \tau f(N_p(u^n)) \left(K_p^{n,n+1}\right)^{\frac{1}{3}}$$

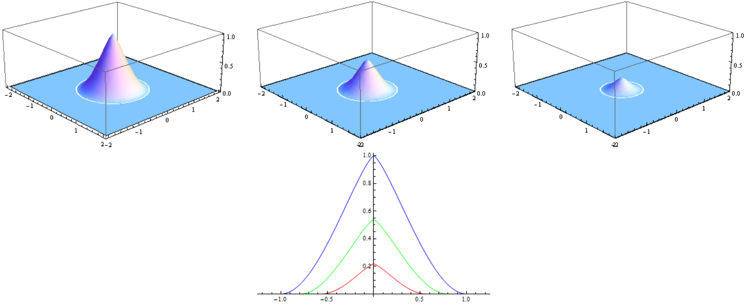
Cranck-Nicolson scheme

$$u_p^{n+1} - \frac{\tau}{2} f(N_p(u^{n+1}))(K_p^{n+1,n+1})^{\frac{1}{3}} = u_p^n + \frac{\tau}{2} f(N_p(u^n))(K_p^{n,n})^{\frac{1}{3}}$$

This nonlinear algebraic system we can compute in iterative way [HM]

Numerical experiments and comparison

Exact solution Carlini Ferreti [CF]
$u(x,y,t) = \max\{1 - (x^2+y^2)^{\frac{2}{3}} - \frac{4}{3}t, 0\}^2$ and homogeneous Dirichlet boundary condition, $\Omega = [-2,2] \times [-2,2]$ and $I = [0, 0.4]$.



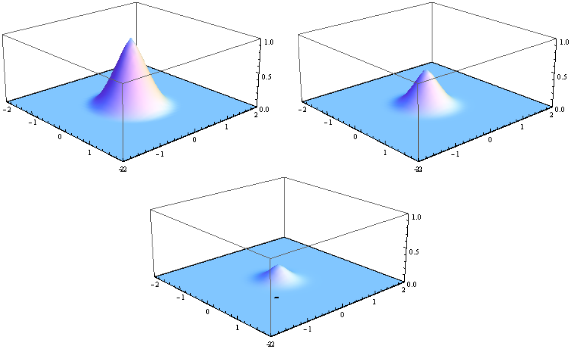
Initial condition (top left) and exact solution at time T=0.2 (top right), exact solution at time T=0.4 (bottom left) and the shape of cuts ($y = 0$) for initial condition (blue) and exact solution at time T=0.2 (green), T=0.4 (red)

Numerical results

N	τ_{EX}	E_2^{EX}	EOC	τ_{SI}	E_2^{SI}	EOC
20	1.0e-02	8.23e-03	-	4.0e-02	3.350e-02	-
40	2.50e-03	2.43e-03	1.762	1.0e-02	1.41e-02	1.307
80	6.25e-04	7.85-04	1.629	2.5e-03	5.40e-03	1.390
160	1.56e-04	3.29e-04	1.257	6.25e-04	1.95e-03	1.474
320	3.92e-05	1.64e-04	1.006	1.56e-04	6.89e-04	1.498
N	τ_{FI}	E_2^{FI}	EOC	τ_{CN}	E_2^{CN}	EOC
20	4.0e-02	1.50e-02	-	5.0e-02	2.49e-02	-
40	1.0e-02	4.45e-03	1.749	2.5e-02	1.25e-02	0.987
80	2.5e-03	1.42e-03	1.650	1.25-02	3.35e-03	1.906
160	6.25e-04	5.08e-04	1.482	6.25e-03	7.48e-04	2.161
320	1.56e-04	2.05e-04	1.306	3.125e-03	1.91e-04	1.972

Numerical results obtained using Crank-Nicolson scheme

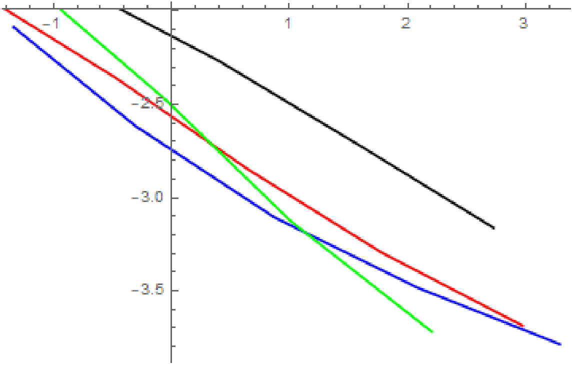
$T = 0; 0.2; 0.4$ for $N = 40$.



Initial condition (top left) and numerical solution at time T=0.2 (top right), numerical solution at time T=0.4 (bottom)for Crank-Nicolson scheme

Comparison error versus CPU time

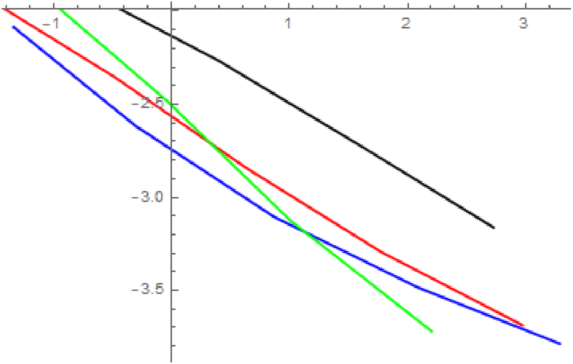
Results in Log Log scale on Figure for all presented schemes



Comparison of error versus CPU time for explicit (blue), semi implicit (black), fully implicit (red) and C-N scheme (green) in Log Log scale

Comparison error versus CPU time

Zoom focused to the smaller error.



Comparison of error versus CPU time for explicit (blue), semi implicit (black), fully implicit (red) and C-N scheme (green) in Log Log scale

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Thank you for your attention!