Towards finite volume methods for the cardiac micromodel

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Abstract

The cardiac micromodel is the system of PDEs that models the electrical behavior of a network of individual cardiac cells, at a microscopic level. It has to be studied in the context of sudden cardiac death (SCD), because several studies have shown that very localized defects in the cell-by-cell structure, or cell-to-cell connections, may induce large scale arrhythmic events such as SCD.
I will explain the model, which consists, for N cells, in $N+1$ Laplace equations in $N+1$ subdomains, the N cells and the extracellular media. These cells are separated from the extracellular media by their membranes, and one with another by connections called gap junctions (GJ). Across these membranes and GJs, the electrical conductivity coefficient is discontinuous, the flux of current is continuous, but the electrical field has a jump, called transmembrane or GJ voltage. These voltages solve systems of nonlinear ordinary differential equations, that couple the voltages, the currents, and additional local state variables. Overall, the system is a Laplace equation in a domain with many interfaces between subdomains, with non standard time-dependent and nonlinear transmission conditions, coupled to ODEs.
I will briefly recall some elements used to prove the well-posedness of the equations. Afterwards, I will explain how we can write the TPFA for this application, and give the first results of numerical analysis that we currently achieved with this scheme.
TPFA methods will not be usable in practice because we are not able to build regular enough meshes of these complex geometries, especially in 3D. I will discuss possible extensions using polynomial interpolation approaches.
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