Very high order finite volume methods for a reaction-diffusion problem, and a parallel implementation

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Abstract

Numerical simulation of the electrical activation of the heart relies on solving a system of reaction-diffusion equation. It is a very demanding application because the solution is composed of sharp wave fronts propagating at high speed. Many practitioners have to use very fine meshes and small time steps in order to capture the phenomena, with standard numerical schemes.

We propose and explore the interest of very high-order cell-centered finite volumes schemes specifically designed for this application, or more generally for reaction diffusion equations. For such time-dependent problems, the reaction terms are usually treated explicitly in time, and thus limit the time step. In many cases, the diffusion term is treated implicitly in time, because the corresponding constraint on the time step may be even more limiting on very fine meshes. We expect to be able to use coarse enough meshes with very high order finite volume methods, in such a way that fully explicit time-stepping methods also become favorable.

I will present this finite volume method, which is based on stencil-based polynomial interpolation, and explain how the scheme may be proved to converge with the expected order of accuracy. In particular, a specific additional mesh regularity assumption is needed, though it does not seem to impair the practical applicability of the method.

Afterward, I will present a domain decomposition technique that we used to design a parallel implementation of the method. A particular attention had to be paid to maintain the scalability in parallel. We propose to constrain the reconstruction stencils inside the subdomains or their first layer of neighbors. A 2D OpenMP based implementation showed that the method keeps its order of accuracy, and scale perfectly up to the level where there are not enough cells in the subdomains. We are currently working on a 3D MPI based implementation, in order to reach larger meshes, and to be able to test 3D cases.

Numerical results prove the order of accuracy of the scheme on simple test cases, the importance of considering very high-order schemes, even for classical tests such as the propagation of planar or spiral waves. I will also show some accuracy and scaling test done with our current parallel implementation.