Finite Volume Method for Level Set Equations on Polyhedral Meshes

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Abstract

We propose a cell-centered finite volume method to numerically solve the G-equation model in premixed turbulent combustion. The governing equations are described by the Favre mean and its fluctuation of the G field which is an implicit function whose zero level set represents the thin flame surface. The equations of the Favre mean and its fluctuation of the G field are commonly called as the G-equation and G-variance equation, respectively. Two equations are coupled and closed by the turbulent flame surface equations and the turbulent flame speed formulations.

The basic form of the G-equation is a standard level set equation containing the advection, normal speed, and mean curvature terms. Compared to conventional algorithms to solve level set equations on a polyhedron mesh, the proposed scheme has mainly three advantages. The first is that it numerically shows higher order of convergence not only on a hexahedron mesh but also on a polyhedron mesh in 3D. The second is that the proposed method can be applicable to the simplest decomposed domains, that is, 1-ring face neighborhood structure, for parallel computation. The third is that a time step restriction caused by the CFL condition is reduced by the proposed semi-implicit scheme.

Well-known approaches to avoid the restriction of the time step are implicit or semi-implicit time discretization methods. An inflow-implicit outflow-explicit (IIOE) method for the discretization of an advective and a normal flow is proposed and used here. The IIOE methods proved to work well for structured grids, but it has been an open issue how to implement them efficiently also for polyhedron meshes. The inflow-based gradient (IBG) IIOE method is derived for polyhedral meshes in order to solve normal flows. The mentioned first and second advantages are observed in the IBG IIOE method. In the G-equation, we extend the algorithm to be used for the advection and mean curvature terms on a polyhedron mesh.

The G-variance equation has the same advection term as the G-equation and we apply the same algorithm used in the G-equation. The G-variance equation also has a turbulent transport term which does not allow turbulent diffusion normal to the mean flame front. It can be approximated as a tangential diffusion term.

A higher order of convergence in numerical examples on polyhedron meshes is presented in the case of each velocity term in the G-equation. Many examples to show the advantages of the proposed method are illustrated.

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