Analysis of a domain decomposition method for a convected Helmholtz like equation

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Abstract

In this work, we are interested in solving an convected Helmholtz like equation. This type of equation occurs in several contexts, for aeroacoustic [1], in the wave-ray method [2] or in a minimisation for solving non linear Schrödinger equation [3].
For the convected Helmholtz equation, it is well-known that using an appropriate change of variables, one can reformulate the equation as a "classical" Helmholtz equation. In particular, this enables to derive Perfectly Matched Layers (PML) formulations [1] or Absorbing Boundary Conditions (ABC) [4]. For domain decomposition methods, this change of variables allows to reduce the convergence analysis to the convergence analysis for the Helmholtz case. In particular, one can derive optimised transmission conditions from the optimized transmission conditions for the Helmholtz problem [5]. In this work, we adapt this idea of change of variables to derive a PML formulation of a slightly more general equation than the convected Helmholtz equation (which covers the three motivating examples). Also, it allows us to make the convergence analysis of a domain decomposition method. In particular, we focus on the effect of using PML or not. Several numerical illustrations will be shown.

1. Stable Perfectly Matched Layers with Lorentz transformation for the convected Helmholtz equation, Marchner P. et al, JCP (2021)

2. Multi-Level Wave-Ray Method for 2D Helmholtz Equation, Verburg P. et al, (2010)

3. Computation of ground states of the Gross–Pitaevskii functional via Riemannian optimization, Danaila I. et al, SIAM Journal on Scientific Computing (2017)

4. Prandtl-Glauert-Lorentz based Absorbing Boundary Conditions for the convected Helmholtz equation, Baruch H. et al, preprint, (2021)

5. Optimized Schwarz methods with overlap for the Helmholtz equation, Gander M. et al, SIAM Journal on Scientific Computing (2016)