

# The parallel full-approximation-scheme in space and time for electromagnetic transient simulation

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1. Introduction
2. DC microgrid model
3. Results
4. Future Work

# Part I

## Introduction

- ▶ Parallel-full-approximation scheme in space and time (PFASST)
- ▶ PinTSimE project

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<sup>1</sup> M. Emmett, M. L. Minion, Toward an efficient parallel in time method for partial differential equations,

Commun. Appl. Math. Comp. Sc. 7, no. 1, 105–132, 2012

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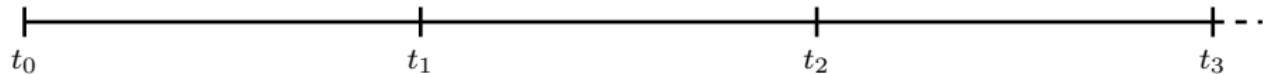
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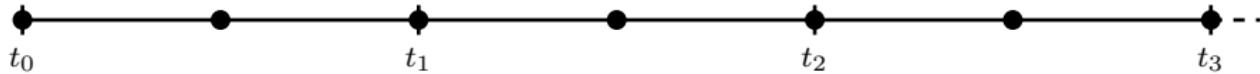
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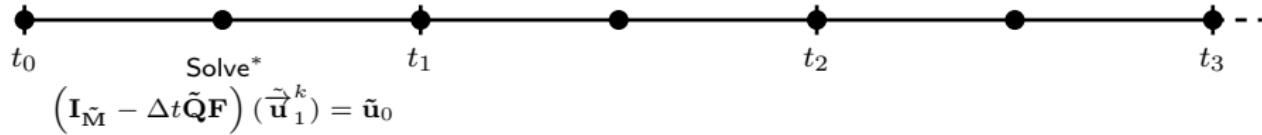
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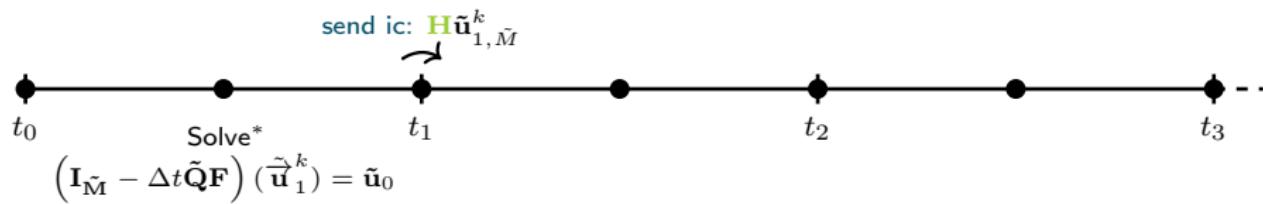
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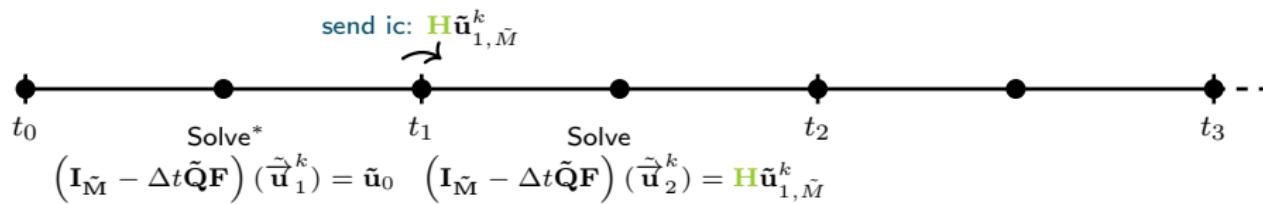
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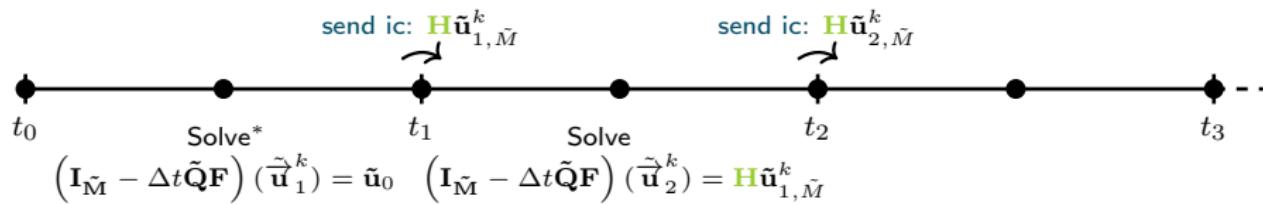
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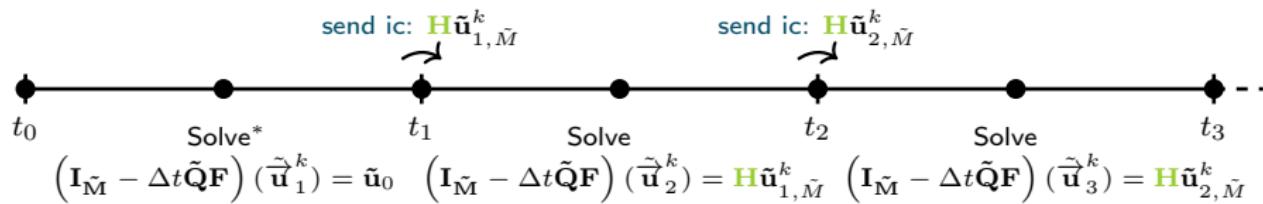
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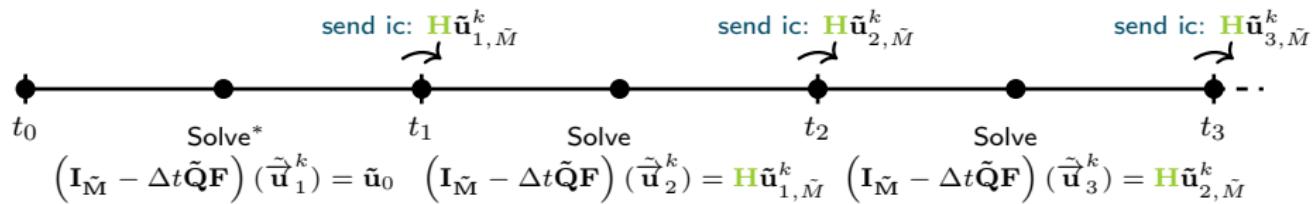
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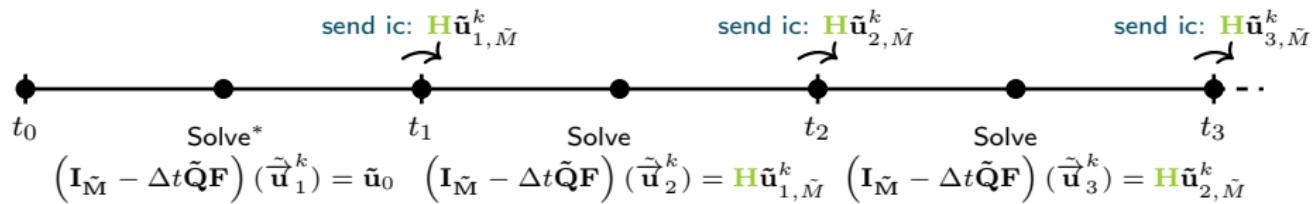
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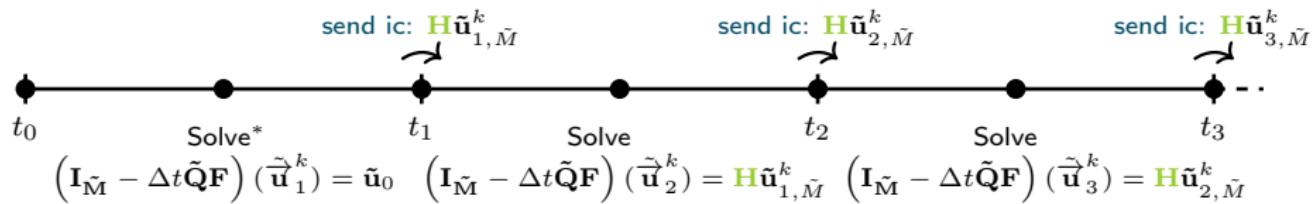


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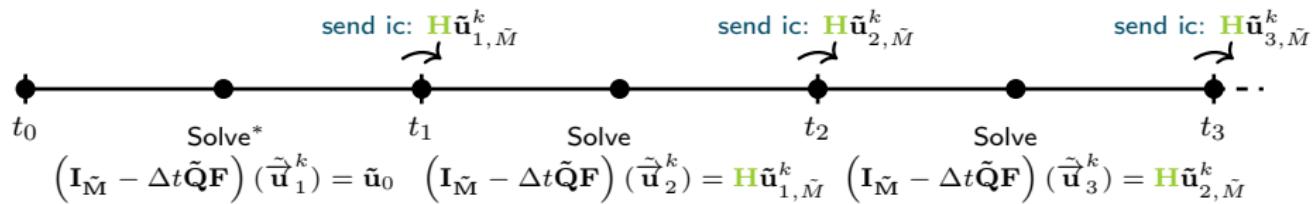


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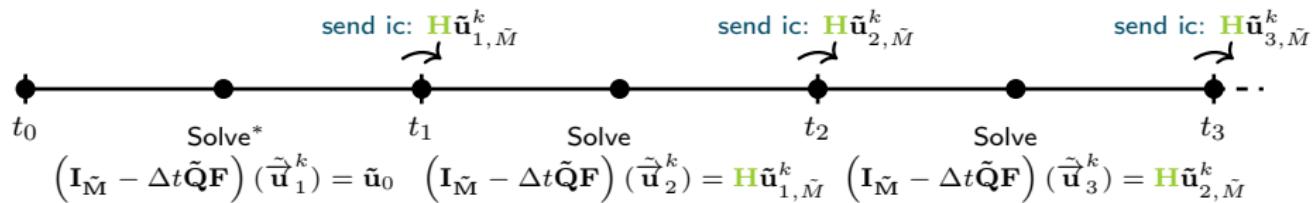


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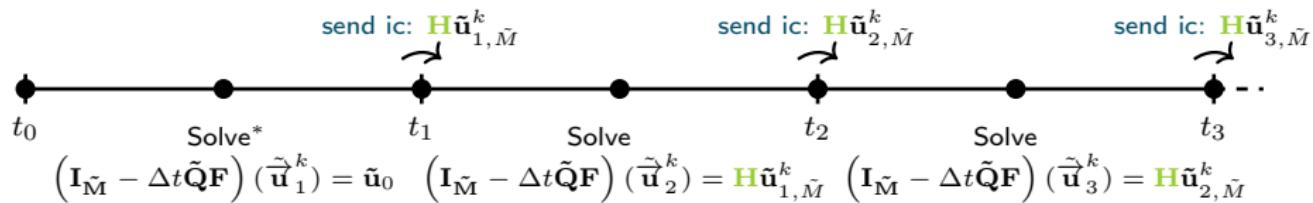
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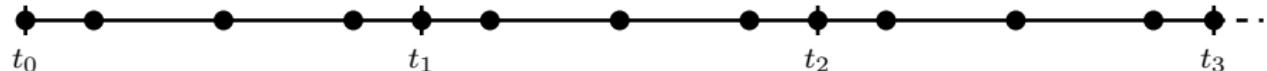
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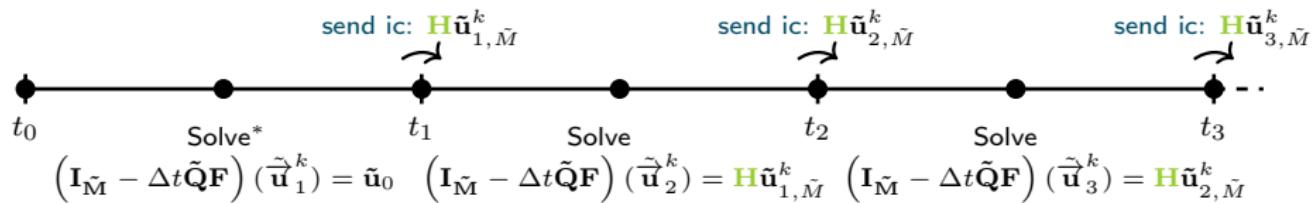
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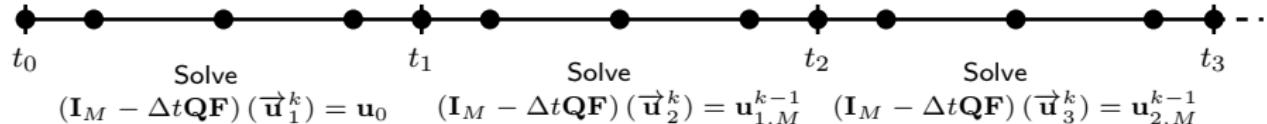
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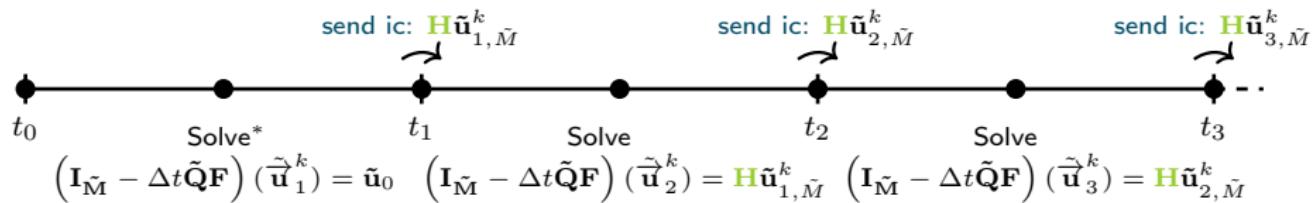
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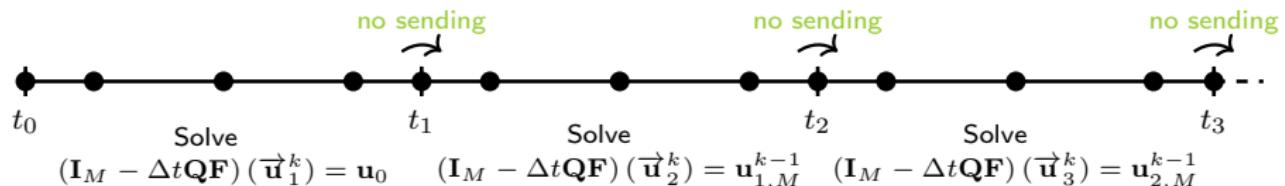
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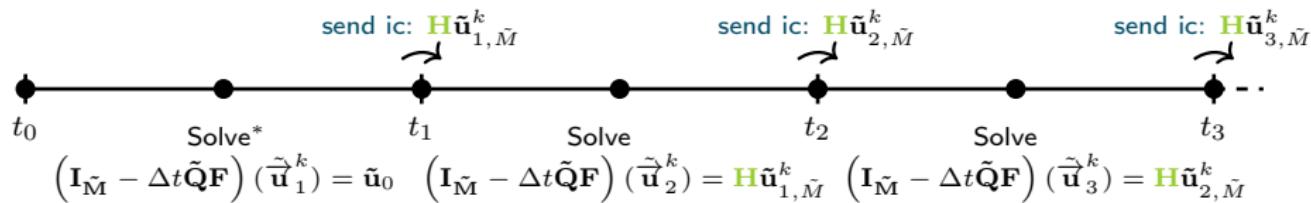
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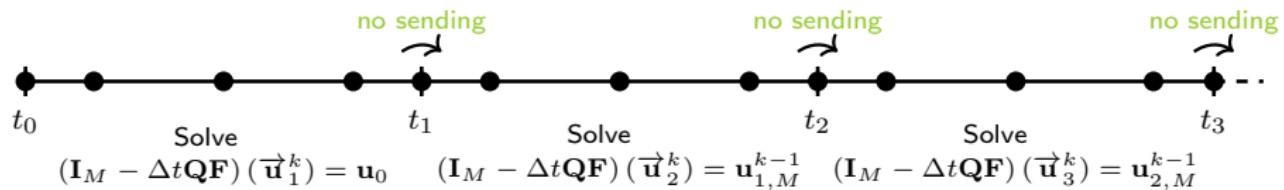
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Gauß-Seidel on coarse level (serial):  $P_{GS}(\vec{\mathbf{u}}) = (\mathbf{I}_{LM} - \mathbf{I}_L \otimes \Delta t \mathbf{Q}_\Delta \mathbf{F} - \mathbf{E} \otimes \mathbf{H})(\vec{\mathbf{u}})$



Jacobi on fine level (parallel):  $P_J(\vec{\mathbf{u}}) := (\mathbf{I}_{LM} - \mathbf{I}_L \otimes \Delta t \mathbf{Q}_\Delta \mathbf{F})(\vec{\mathbf{u}})$



\*: a tilde marks values on coarse level

<sup>2</sup>: M. Bolten, D. Moser, R. Speck, A multigrid perspective on the parallel full approximation scheme in space and time, Numerical Linear Algebra with Applications 24, 6, 2017

# PFASST Algorithm<sup>1,2</sup>

1. Execute Jacobi solver on fine level in parallel using  $P_J$

# PFASST Algorithm<sup>1,2</sup>

1. Execute Jacobi solver on fine level in parallel using  $P_J$
2. Compute  $\tau$ -correction

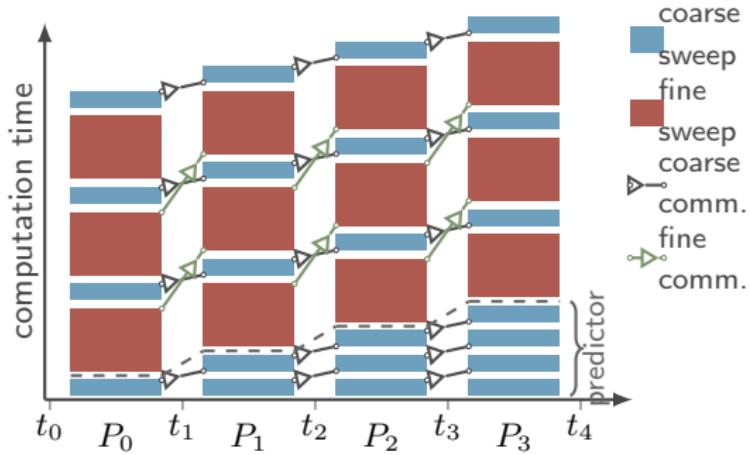
# PFASST Algorithm<sup>1,2</sup>

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# PFASST Algorithm<sup>1,2</sup>

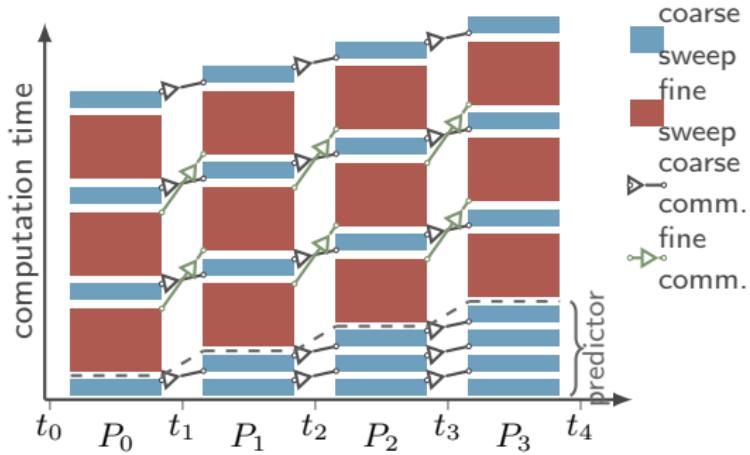
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4. Correct solution

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1. Execute Jacobi solver on fine level in parallel using  $P_J$
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<sup>3</sup>: Fabian Koehler, PFASST tikz, <https://github.com/Parallel-in-Time/pfasst-tikz>, 2015

## Members:



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(BUW)



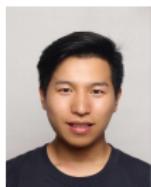
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- ▶ PinTSimE - Parallel-in-Time Simulation in multimodal Energy systems

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- ▶ Multimodal - Energy system in which different energy grids are *connected* such as power grid, heat grid, gas grid,...

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- ▶ First focus lies on DC power microgrid

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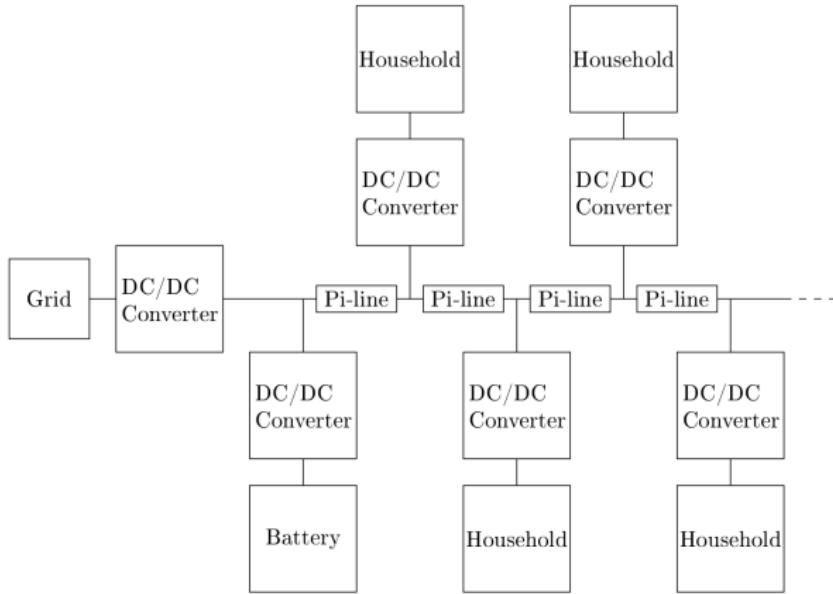
- ▶ PinTSimE - Parallel-in-Time Simulation in multimodal Energy systems
- ▶ Multimodal - Energy system in which different energy grids are *connected* such as power grid, heat grid, gas grid,..
- ▶ First focus lies on DC power microgrid
- ▶ **Goal:** Simulation faster than real-time

## Part II

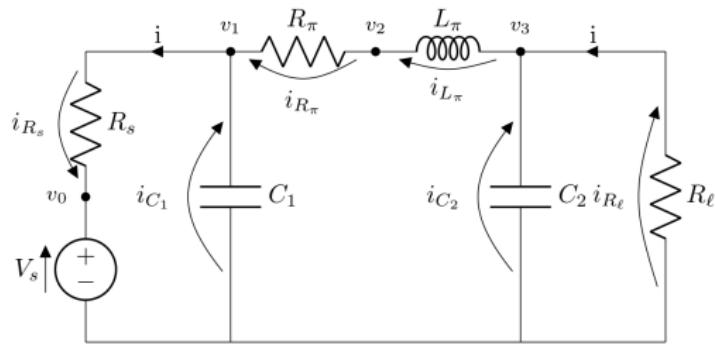
### DC microgrid model

- ▶ Components
- ▶ Switching processes

# DC microgrid

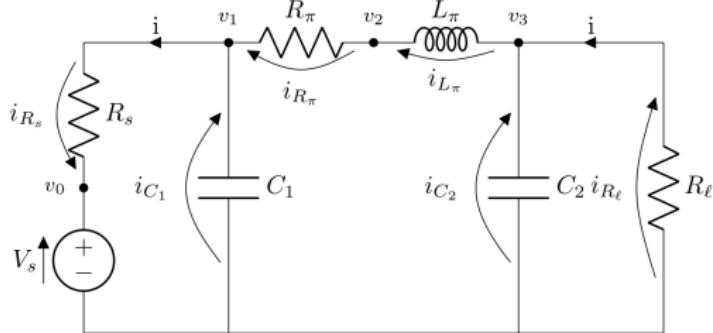


# Pi-line model



Pi-line model

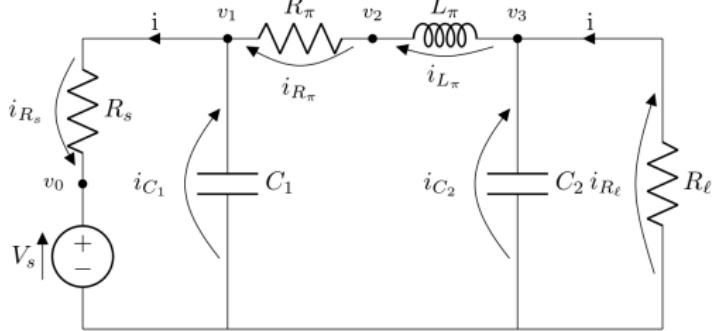
# Pi-line model



Pi-line model

Model can be described as  
ODE System:

# Pi-line model

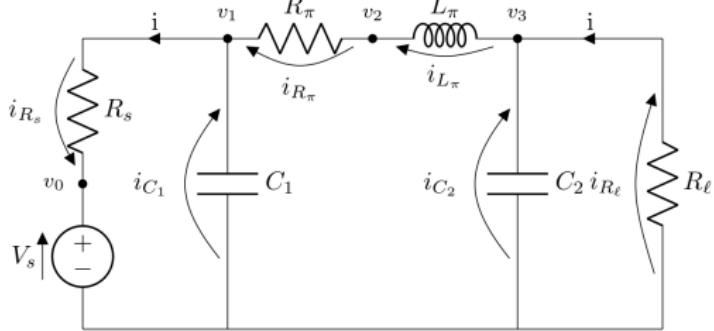


Pi-line model

Model can be described as  
ODE System:

$$\frac{d}{dt} u(t) = Au(t) + f(t)$$

# Pi-line model



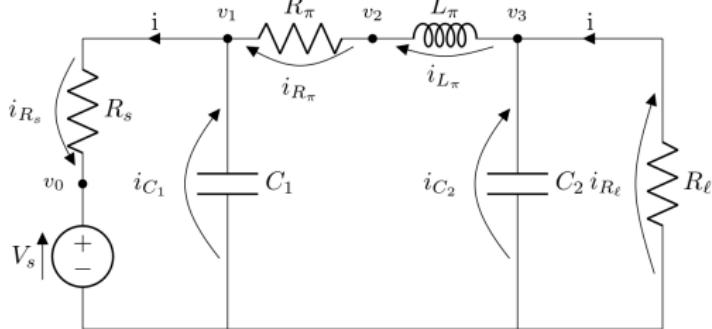
Pi-line model

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Properties:

# Pi-line model



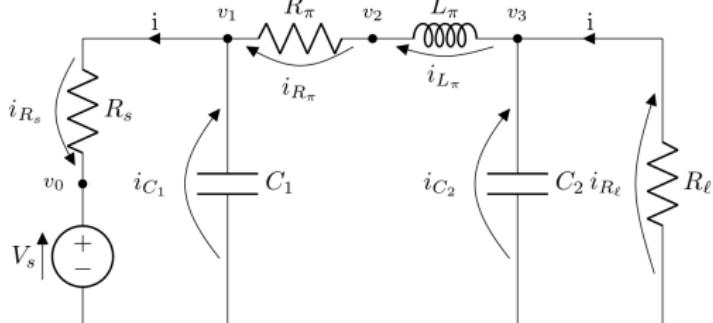
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Model can be described as  
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$$\frac{d}{dt} u(t) = Au(t) + f(t)$$

Properties:

- serves as transmission line in DC microgrid



Pi-line model

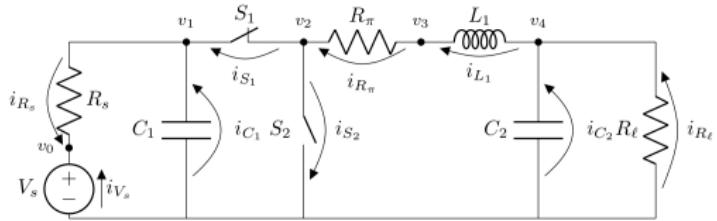
Model can be described as  
ODE System:

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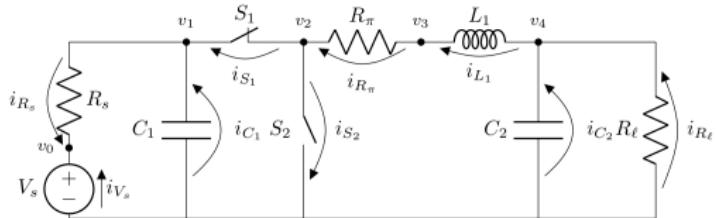
Properties:

- ▶ serves as transmission line in DC microgrid
- ▶ no special events in the simulation

# Buck converter model

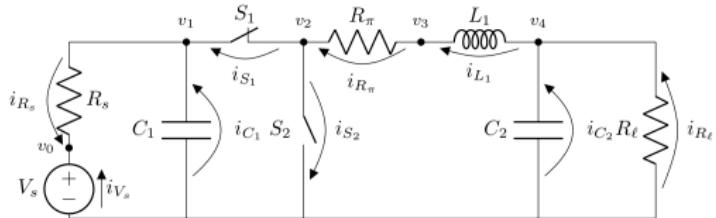


# Buck converter model



Buck converter model with the states

# Buck converter model

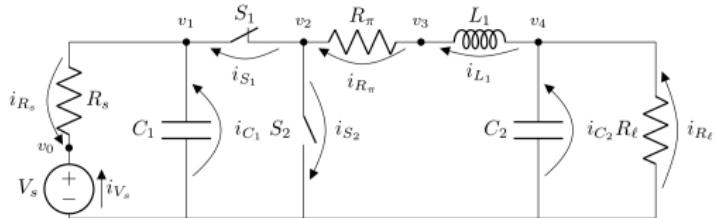


Buck converter model with the states

$$FS = \{S_1 = 1, S_2 = 0\}$$

and

# Buck converter model



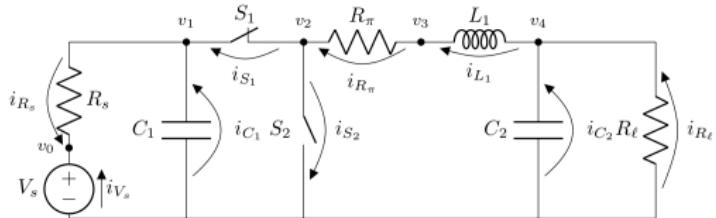
Buck converter model with the states

$$FS = \{S_1 = 1, S_2 = 0\}$$

and

$$SS = \{S_1 = 0, S_2 = 1\}$$

# Buck converter model



Each state can be described  
as ODE System:

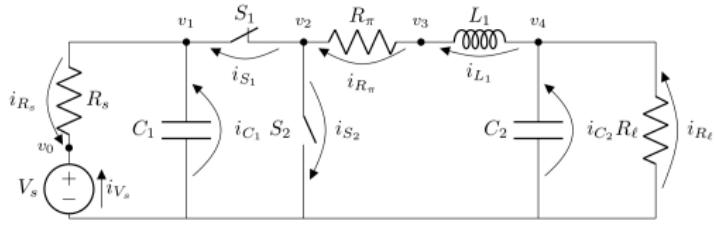
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# Buck converter model



Each state can be described  
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$$\text{FS: } \frac{d}{dt} u(t) = A_1 u(t) + f_1(t)$$

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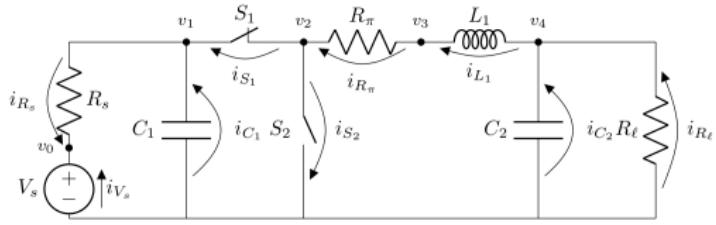
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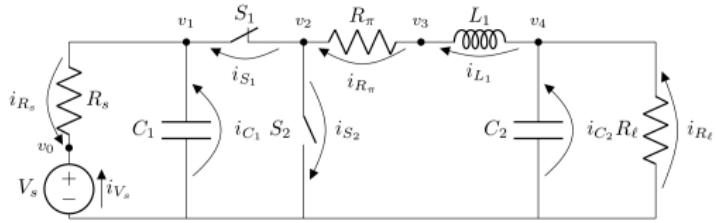
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Properties:

# Buck converter model



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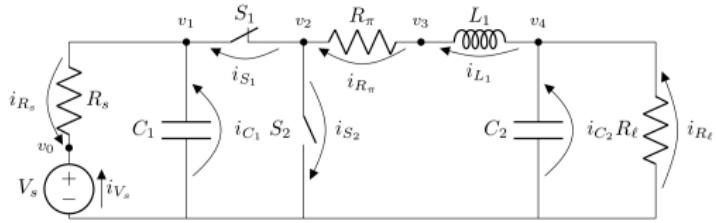
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Properties:

- ▶ converts large input voltage in a smaller output voltage

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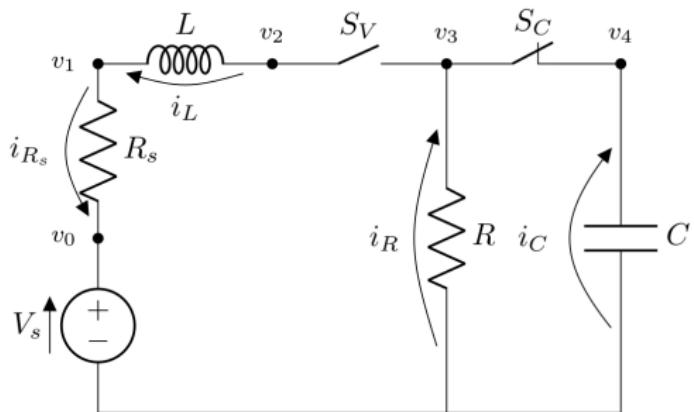
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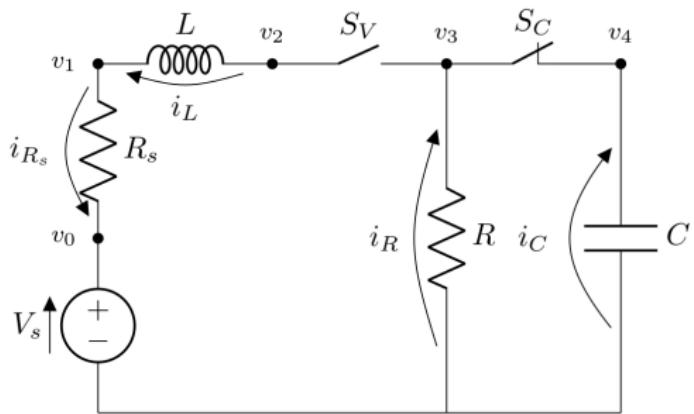
Properties:

- ▶ converts large input voltage in a smaller output voltage
- ▶ switching produces many discrete events

# Battery drain model

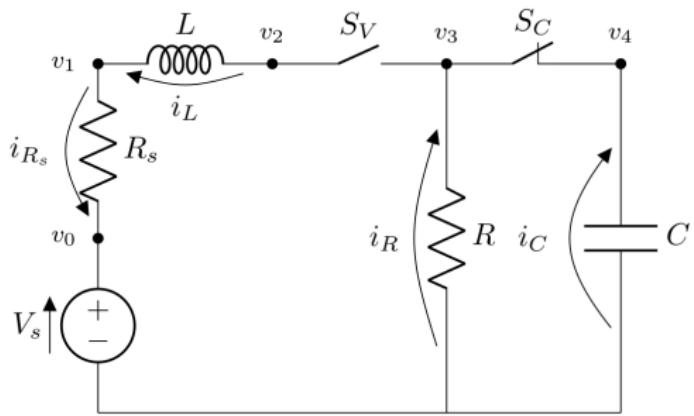


## Battery drain model



Battery model with the states

# Battery drain model

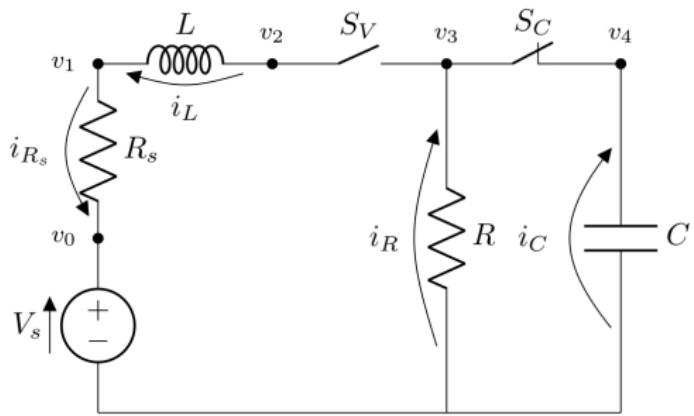


Battery model with the states

$$FS = \{S_C = 1, S_V = 0\}$$

and

## Battery drain model



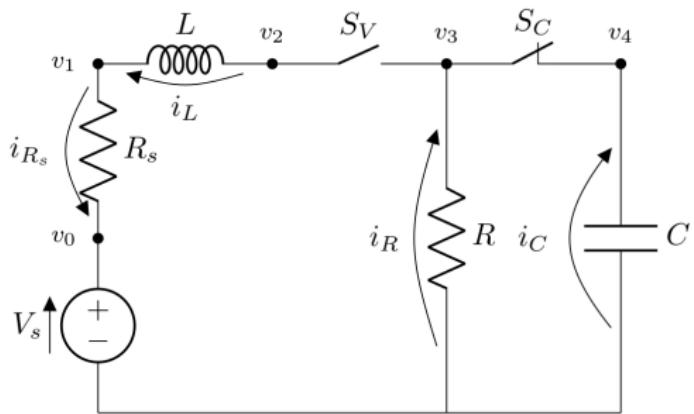
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and

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# Battery drain model



Each state can be described  
as **ODE System:**

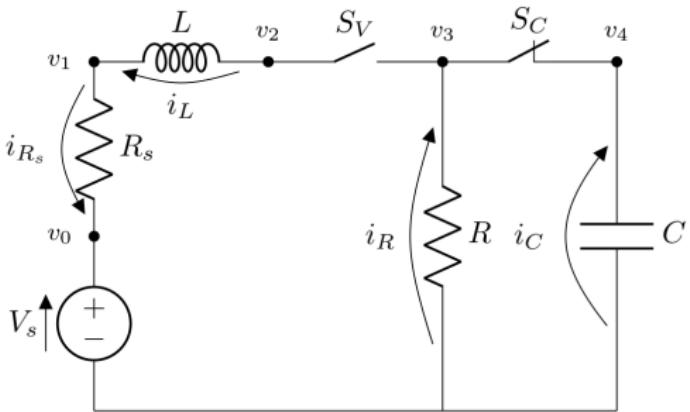
Battery model with the states

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# Battery drain model



Each state can be described as **ODE System:**

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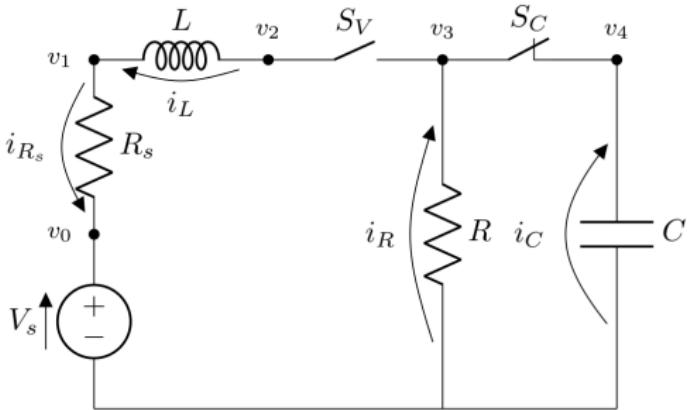
Battery model with the states

$$FS = \{S_C = 1, S_V = 0\}$$

and

$$SS = \{S_C = 0, S_V = 1\}$$

# Battery drain model



Each state can be described as **ODE System:**

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**Properties:**

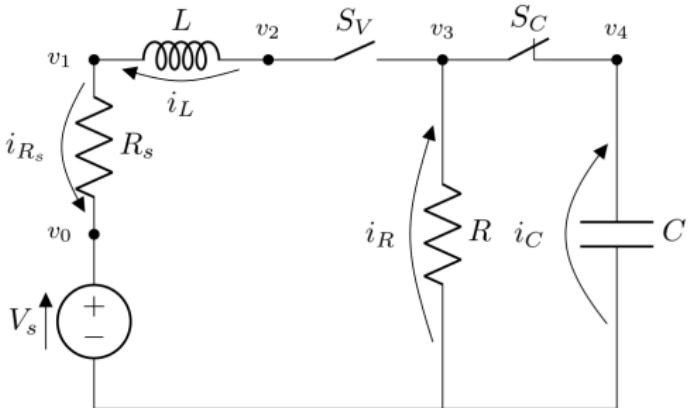
Battery model with the states

$$FS = \{S_C = 1, S_V = 0\}$$

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# Battery drain model



Each state can be described as **ODE System:**

$$FS: \frac{d}{dt} u(t) = A_1 u(t) + f_1(t)$$

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**Properties:**

- ▶ capacitor supplies energy

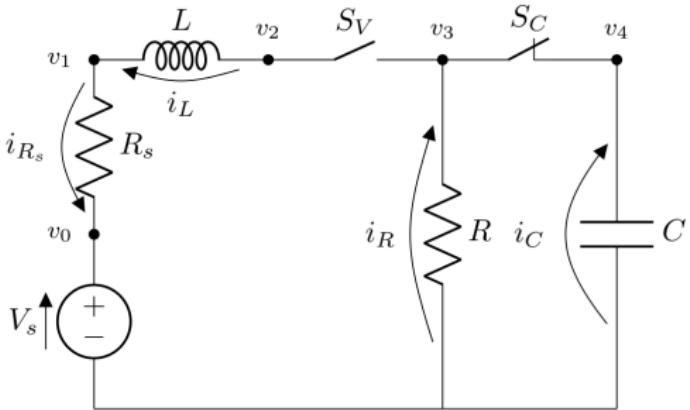
Battery model with the states

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and

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# Battery drain model



Battery model with the states

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Each state can be described as **ODE System:**

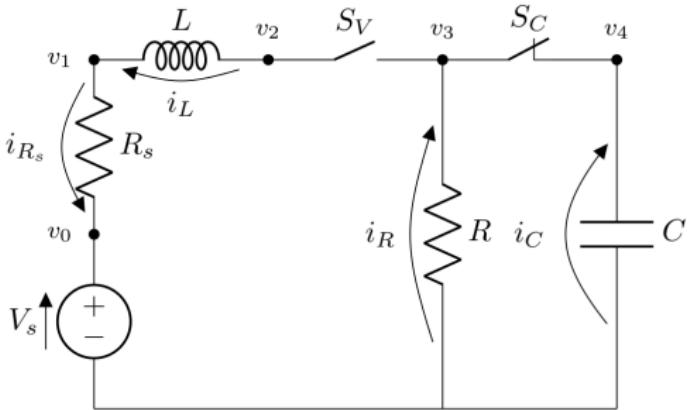
$$FS: \frac{d}{dt} u(t) = A_1 u(t) + f_1(t)$$

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**Properties:**

- ▶ capacitor supplies energy
- ▶ when energy drops below a reference voltage, switch to voltage source

# Battery drain model



Battery model with the states

$$FS = \{S_C = 1, S_V = 0\}$$

and

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Each state can be described as **ODE System**:

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$$SS: \frac{d}{dt} u(t) = A_2 u(t) + f_2(t)$$

**Properties:**

- ▶ capacitor supplies energy
- ▶ when energy drops below a reference voltage, switch to voltage source
- ▶ only one single discrete event

# Switching processes

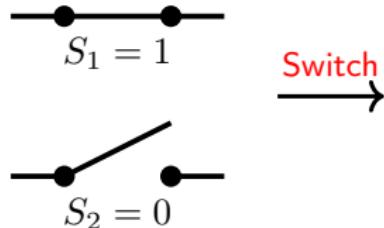


$$S_1 = 1$$

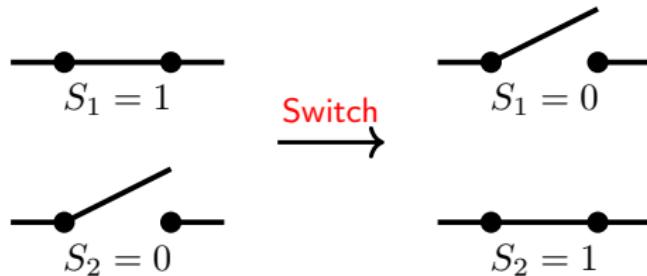


$$S_2 = 0$$

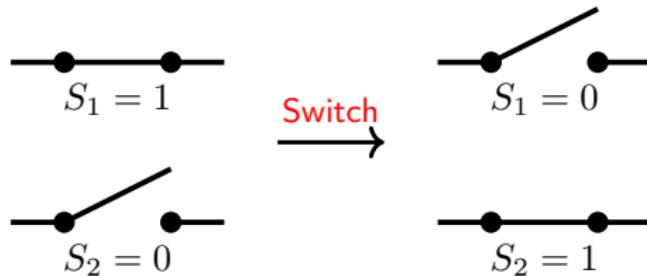
## Switching processes



## Switching processes

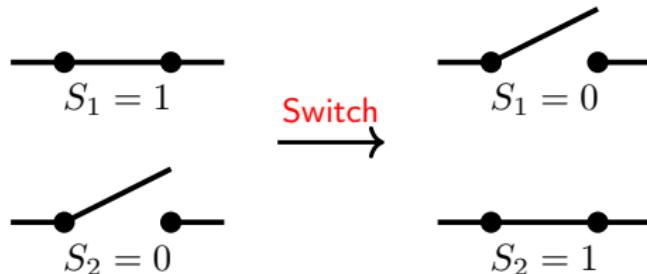


## Switching processes



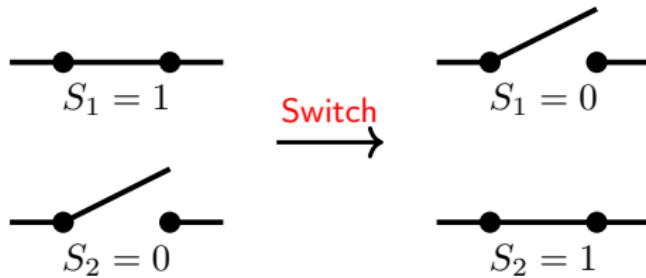
- discrete events

## Switching processes



- discrete events
- for models: switching from one ODE system to another

## Switching processes



- ▶ discrete events
- ▶ for models: switching from one ODE system to another
- ▶ needs to investigate their behavior in SDC (and PFASST)

## Occurence of a switch

Two possibilities, where a switch can occur:

## Occurrence of a switch

Two possibilities, where a switch can occur:

- ▶ on a time step

## Occurrence of a switch

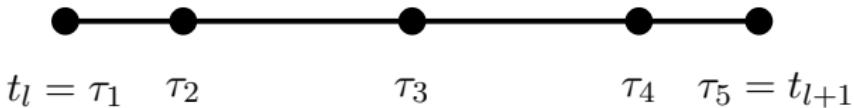
Two possibilities, where a switch can occur:

- ▶ on a time step
- ▶ between collocation nodes

## Occurrence of a switch

Two possibilities, where a switch can occur:

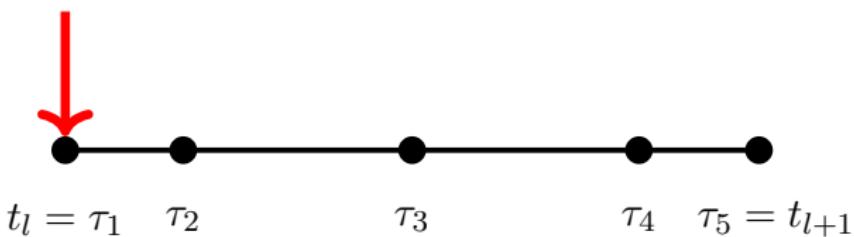
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Two possibilities, where a switch can occur:

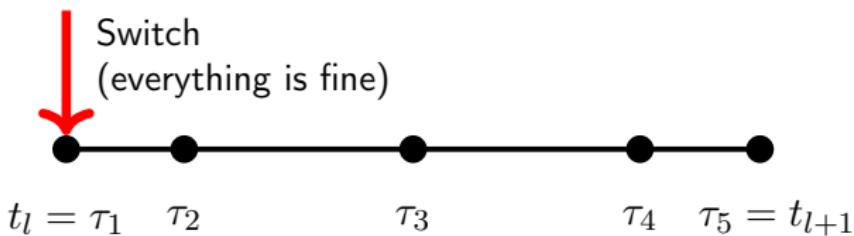
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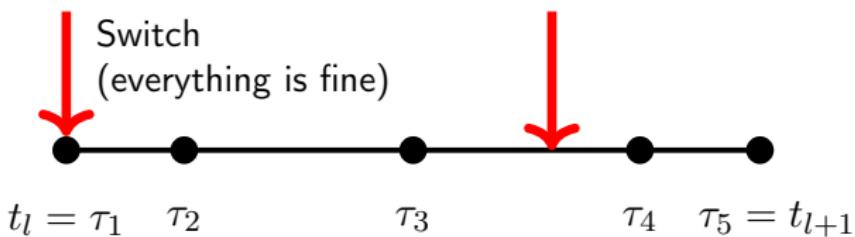
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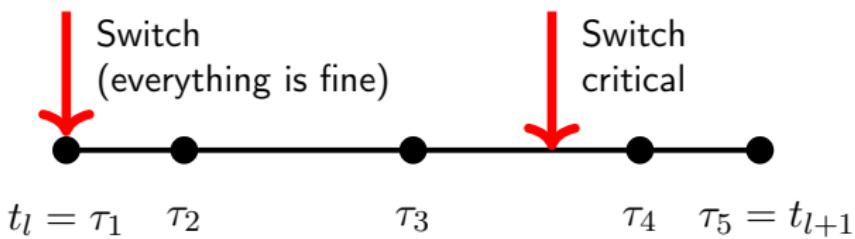
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## Occurrence of a switch

Two possibilities, where a switch can occur:

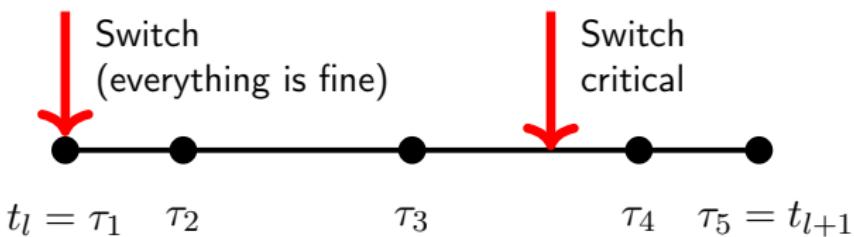
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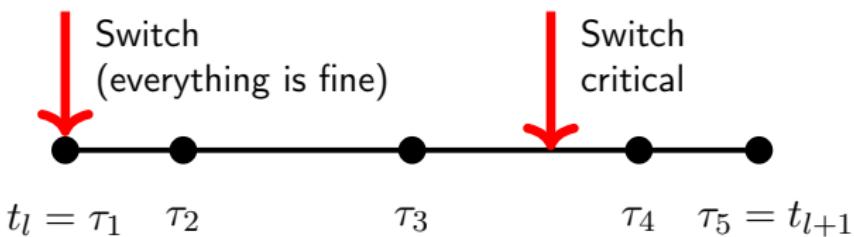


Challenges:

## Occurrence of a switch

Two possibilities, where a switch can occur:

- ▶ on a time step
- ▶ between collocation nodes



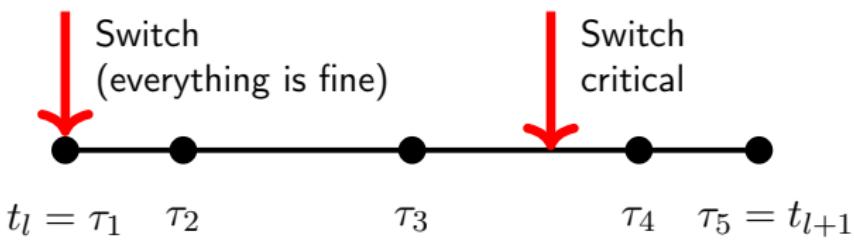
### Challenges:

- ▶ Occurring of switch between collocation nodes leads to later resolution of behavior

## Occurrence of a switch

Two possibilities, where a switch can occur:

- ▶ on a time step
- ▶ between collocation nodes



### Challenges:

- ▶ Occurring of switch between collocation nodes leads to later resolution of behavior
- ▶ Accuracy is worse: energy monitoring is delayed

## Part III

## Results

## Settings:

## Settings:

- ▶  $t_0 = 0, \Delta t = 10^{-2}, T = 15$

## Settings:

- ▶  $t_0 = 0, \Delta t = 10^{-2}, T = 15$
- ▶ IMEX SDC with 5 *Gauss-Lobatto (GL)* nodes used for simulation

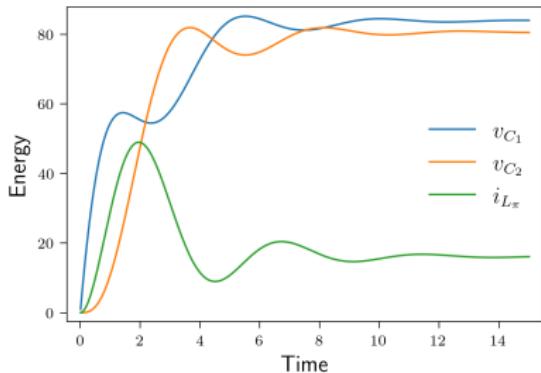
## Settings:

- ▶  $t_0 = 0, \Delta t = 10^{-2}, T = 15$
- ▶ IMEX SDC with 5 *Gauss-Lobatto (GL)* nodes used for simulation
- ▶ local (per subinterval)  $restol = 10^{-13}$

# Pi-line modelling - SDC

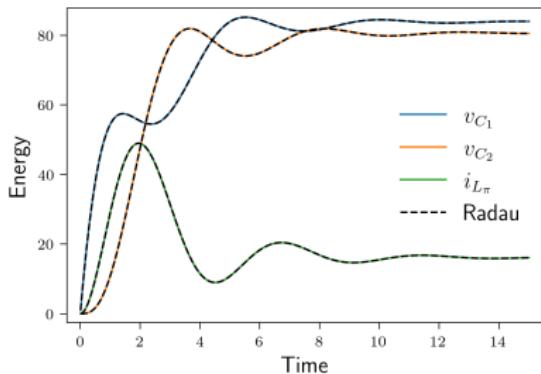
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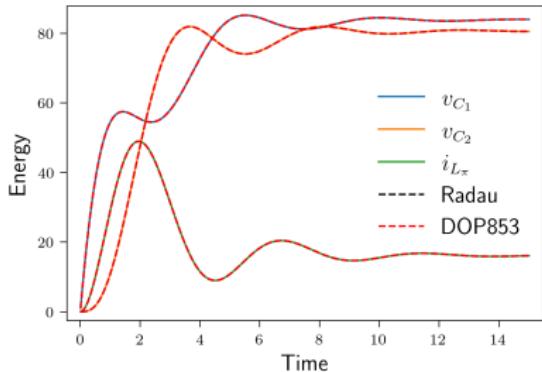
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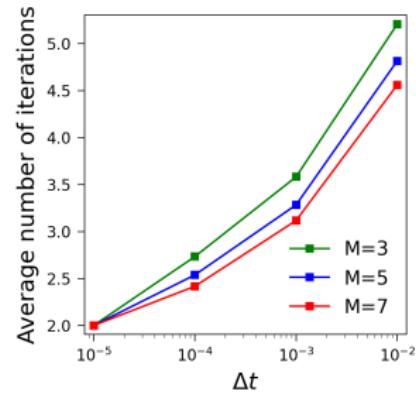
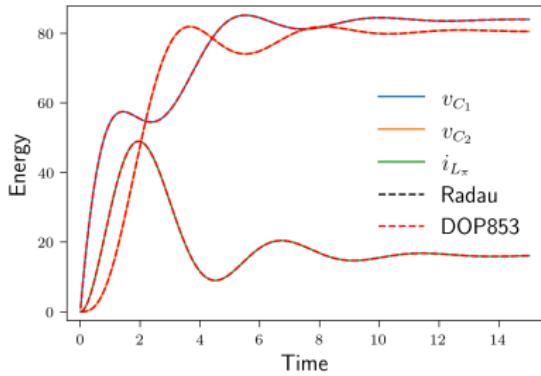
## Settings:

- ▶  $t_0 = 0, \Delta t = 10^{-2}, T = 15$
- ▶ IMEX SDC with 5 *Gauss-Lobatto (GL)* nodes used for simulation
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## Settings:

## Settings:

- ▶ Same as for SDC,  
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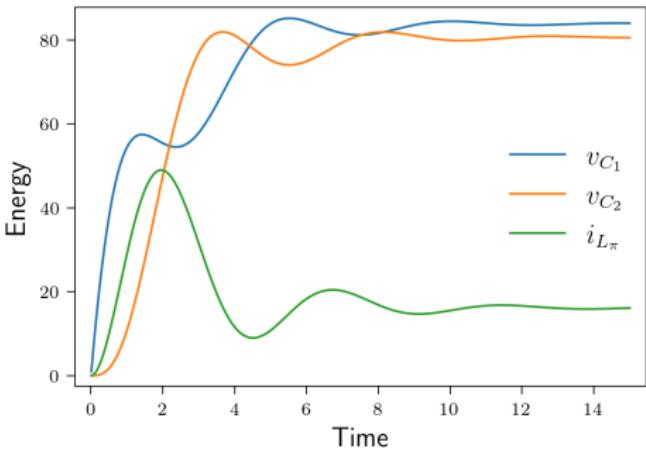
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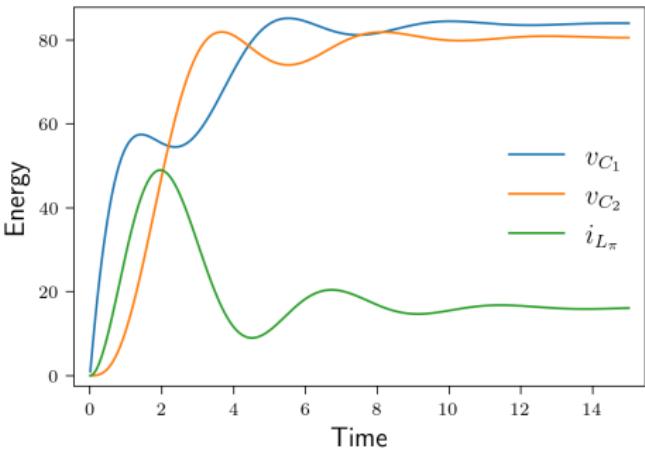
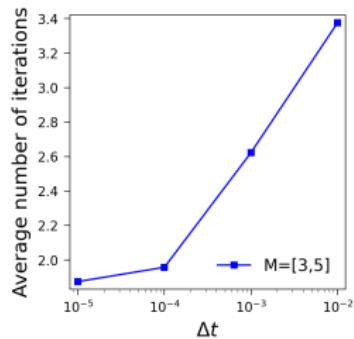
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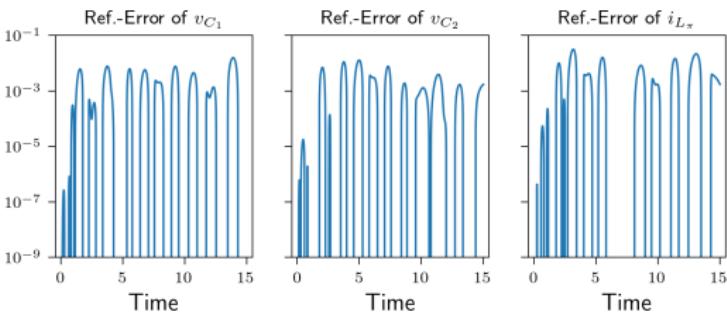
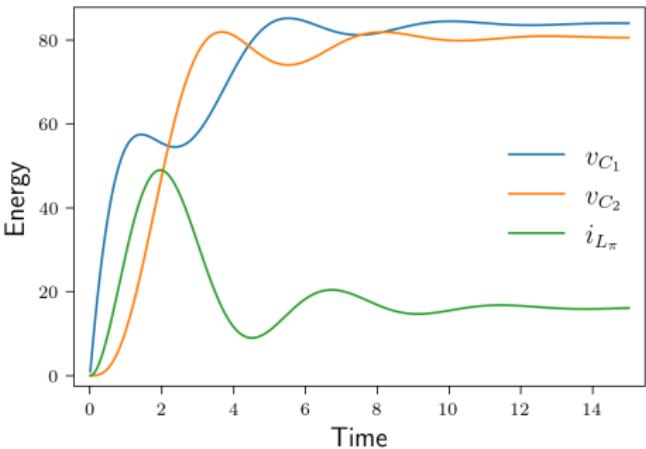
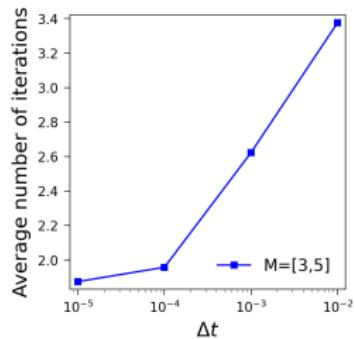
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# Buck converter modelling - SDC

Buck converter manipulation: Implementation of *only one switch!*

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Setting:

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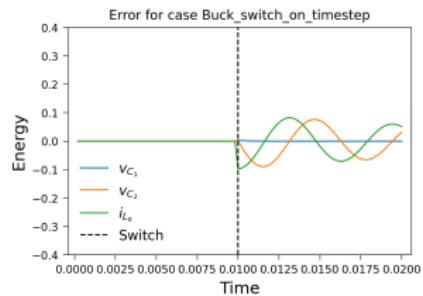
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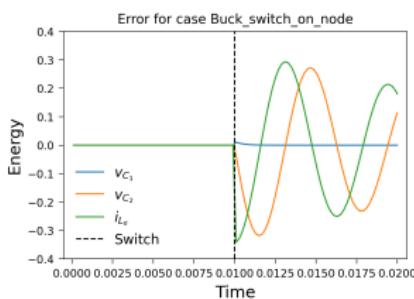
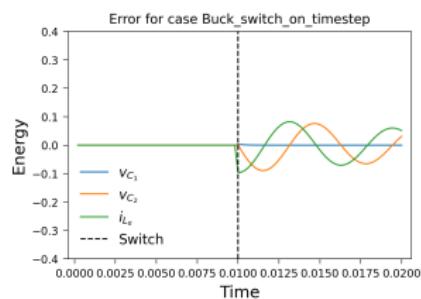
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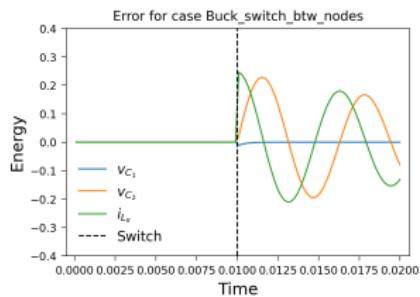
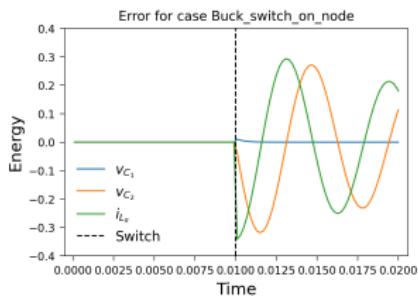
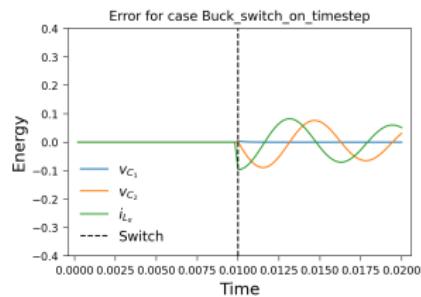
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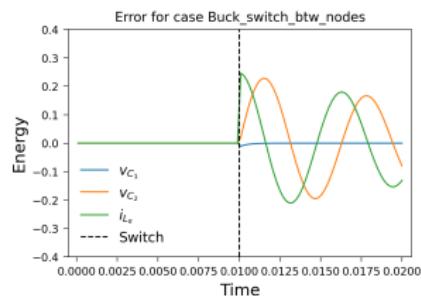
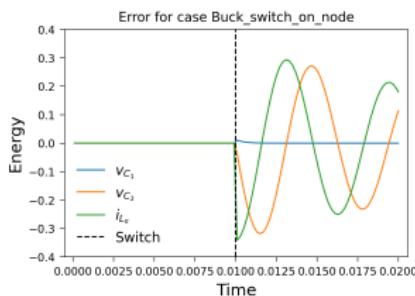
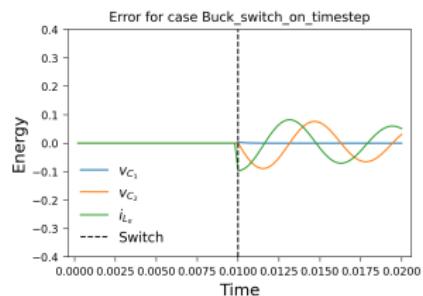
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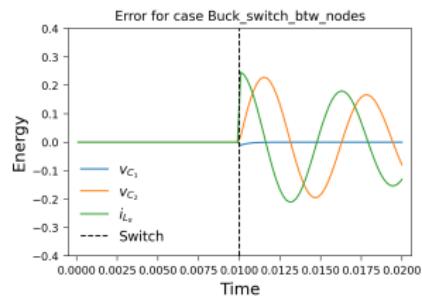
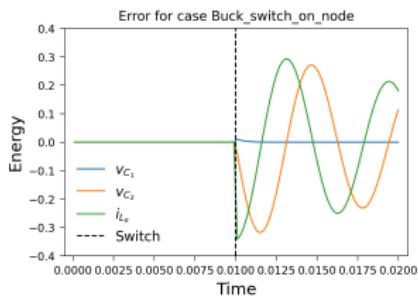
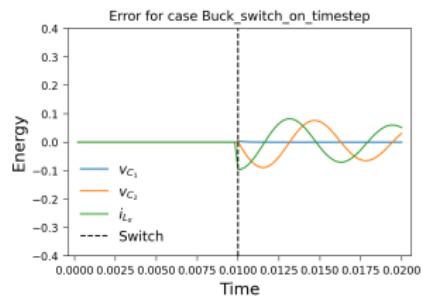
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- Error for case *Switch on timestep* relatively small
- Error for *other cases* show larger error

Battery drain model has *internal switching*: Switching depends on system dynamics

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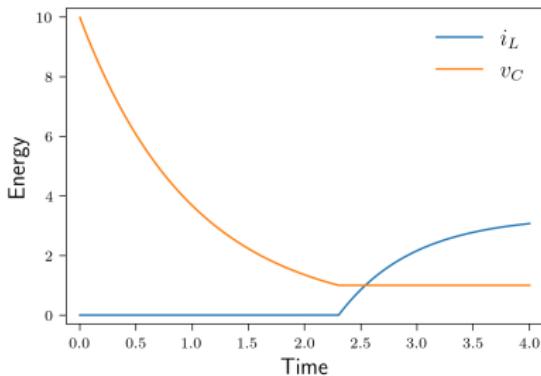
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## Part IV

### Future work

- ▶ Development of switch estimator (first, for SDC - then, for PFASST)

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Any questions or comments to the talk?