# Parallel-in-time optimization of induction motors

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MATHEMATICAL MODELLING, ANALYSIS AND COMPUTATIONAL MATHEMATICS

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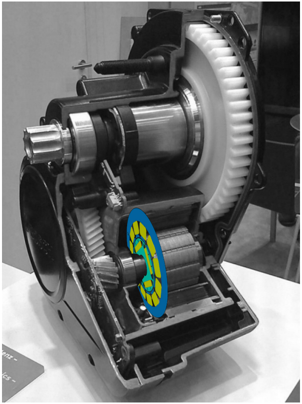
# Modern corporate design of electrical motors



- eBike with a synchronous machine
- Robust geometry optimization
- Expensive time domain simulations

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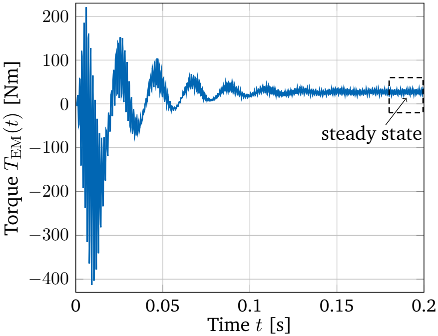
# Time-domain simulation of three-phase induction motors




Time-domain torque evolution

- Solution typically consists of a transient part, followed by a (periodic) steady state
- Steady-state operating characteristics are important design criteria

# The Eddy Current Problem - neglect the displacement current in Maxwell's equations

- Eddy current problem in $\Omega \times (0, T_{\text{end}}]$

$$\sigma \frac{\partial \mathbf{A}}{\partial t} + \nabla \times (\nu \nabla \times \mathbf{A}) = \mathbf{J}_s$$

with suitable BCs and IC $\mathbf{A}(\mathbf{x}, 0) = \mathbf{A}_0(\mathbf{x})$,
$\mathbf{A}$ unknown magnetic vector potential,
$\mathbf{J}_s$ source current density,
$\sigma(\mathbf{x}), \nu(\mathbf{x}, \mathbf{A})$ conductivity and reluctivity

2D cross section of "im_3kw" model

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- Additional equation for three-phase input voltage $\rightsquigarrow$ coupled field-circuit system

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- Semi-discretization in space yields DAE

$$M_\sigma \frac{d\mathbf{a}}{d}(t) + K_\nu(\mathbf{a}(t))\mathbf{a}(t) - X\mathbf{i}(t) = \mathbf{0},$$

$$X^T \frac{d\mathbf{a}}{d}(t) + R\mathbf{i}(t) = \mathbf{v}(t)$$

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2D cross section of "im_3kw" model

Design process requires many simulations
$\rightarrow$ **reduce using PinT**

# Time integration

- Consider system of ODEs

$$\mathbf{u}'(t) = \mathbf{f}(t, \mathbf{u}(t)), \quad t \in (0, T_{\text{end}}], \quad \mathbf{u}(0) = \mathbf{g}_0$$

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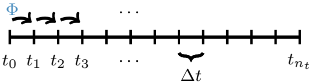
$$\mathbf{u}'(t) = \mathbf{f}(t, \mathbf{u}(t)), \quad t \in (0, T_{\text{end}}], \quad \mathbf{u}(0) = \mathbf{g}_0$$

- Time-stepping problem ("$\Phi$-form")

$$\mathbf{u}_i = \Phi_i(\mathbf{u}_{i-1}), \ \mathbf{u}_0 = \mathbf{g}_0, \ i = 1, \ldots, n_t$$



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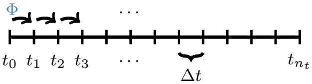
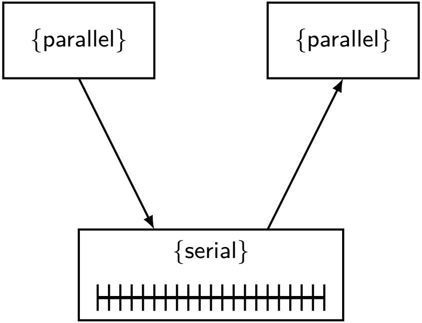



- Equivalent space-time system

$$\mathcal{A}(\mathbf{u}) \equiv \begin{bmatrix} \mathbf{u}_0 \\ \mathbf{u}_1 - \Phi_1(\mathbf{u}_0) \\ \vdots \\ \mathbf{u}_{n_t} - \Phi_{n_t}(\mathbf{u}_{n_t-1}) \end{bmatrix} = \begin{bmatrix} \mathbf{g}_0 \\ \mathbf{0} \\ \vdots \\ \mathbf{0} \end{bmatrix} \equiv \mathbf{g}$$

# Multigrid in time: motivation for AT-MGRIT

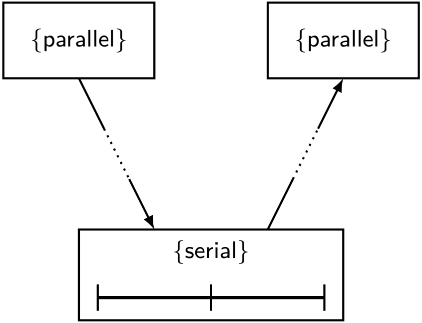
Two-level




⊖ Limited parallelism

⊕ Small time steps on coarsest grid

Multilevel

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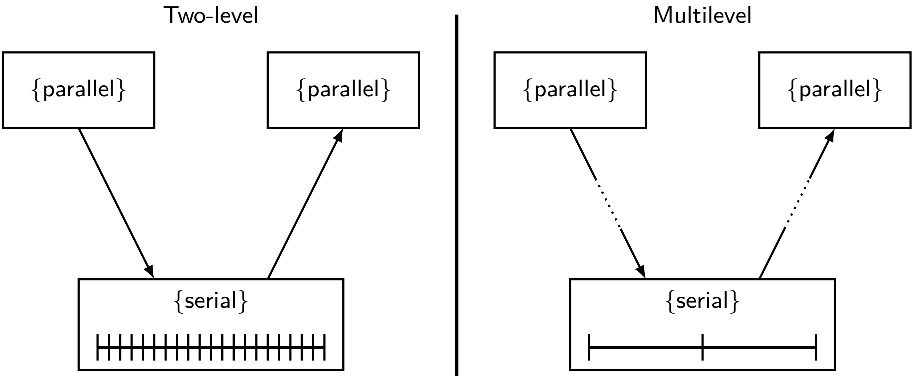


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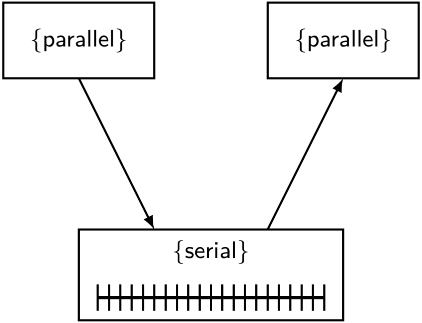


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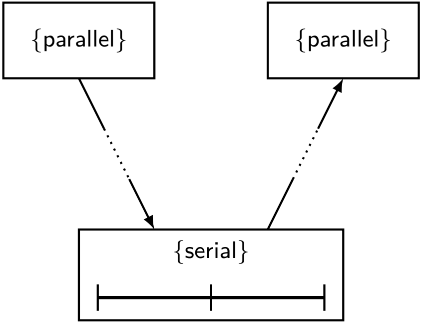


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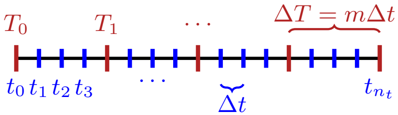
⊖ Large time steps on coarsest grid

- Method with few but still small time steps on the coarsest grid?
- Enable parallelism at the coarsest level?

# Asynchronous Truncated MGRIT (AT-MGRIT)*

- Partition the time grid into C-points and F-points




* J. Hahne et. al. *Asynchronous Truncated Multigrid-reduction-in-time*, arXiv:2107.09596v1.

# Asynchronous Truncated MGRIT (AT-MGRIT)*

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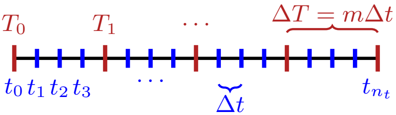
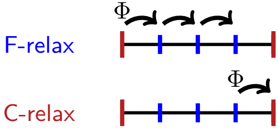
  - alternates between F- and C-points

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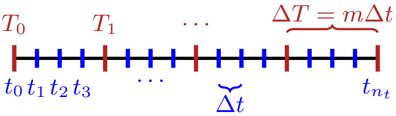


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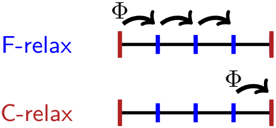
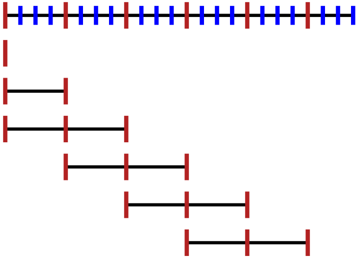



- Truncated local coarse grids
  - One grid per point on coarsest level
  - Based on local grid size $k$ (#points per grid)
  - restriction = injection
  - interpolation = injection (1 point) + F-relax



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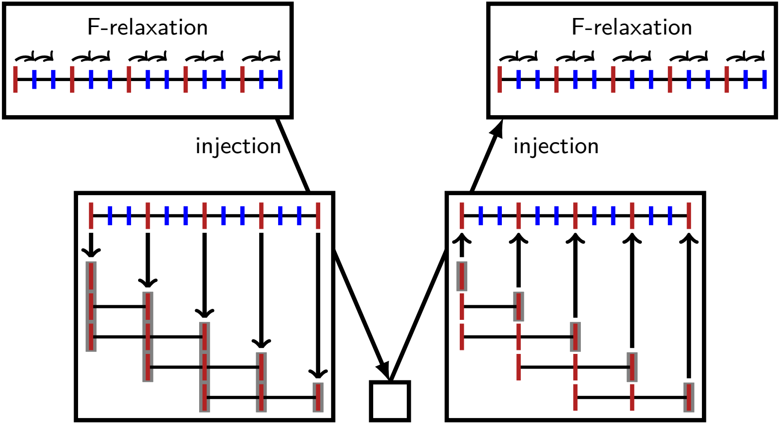
# Two-level AT-MGRIT




- Asynchronous: Communication only required for computation and distribution of the residuals

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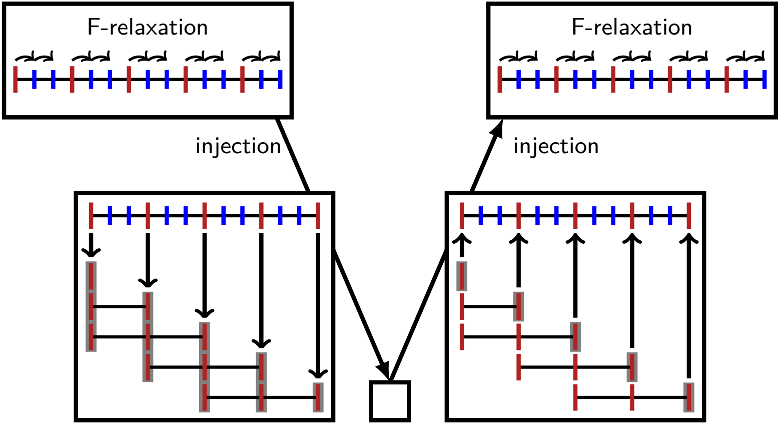


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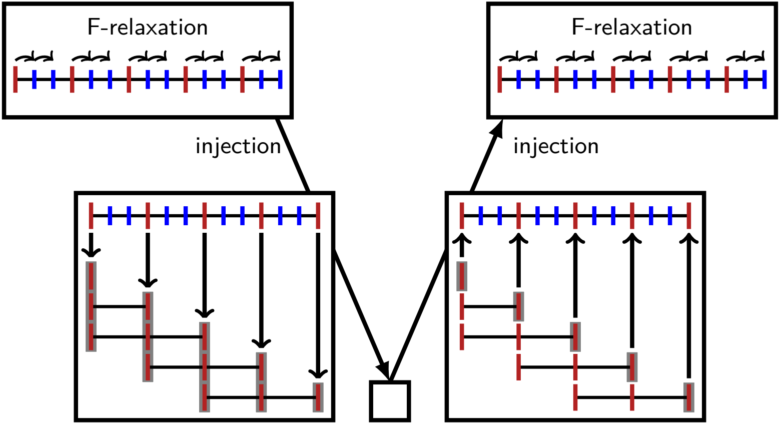


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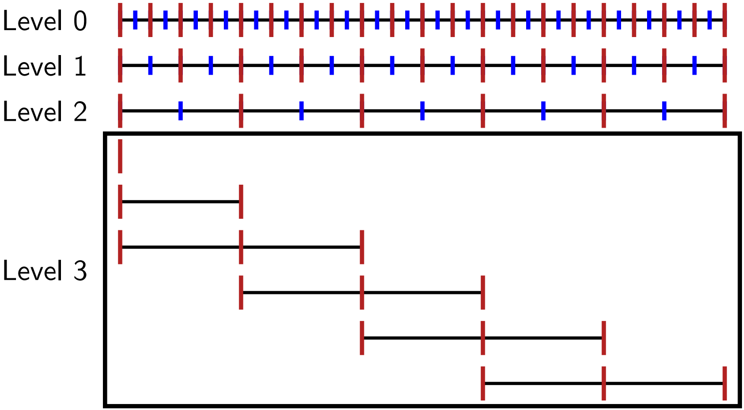
- Extends to nonlinear problems with FAS formulation
- Equivalent to Parareal if $k = \#$C-points

# Multilevel AT-MGRIT



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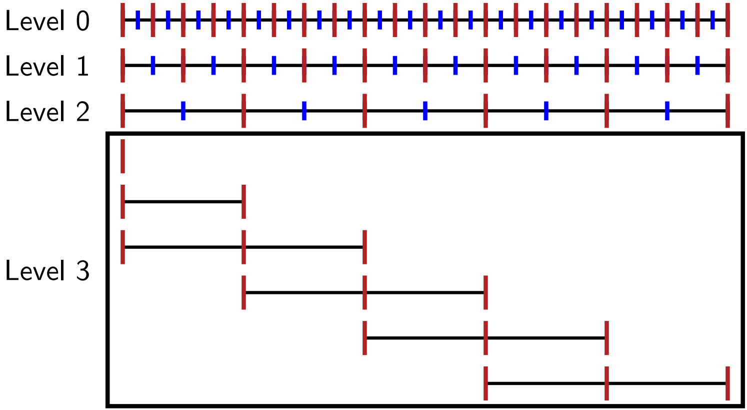


- Equivalent to MGRIT if $k = \#$points coarsest level

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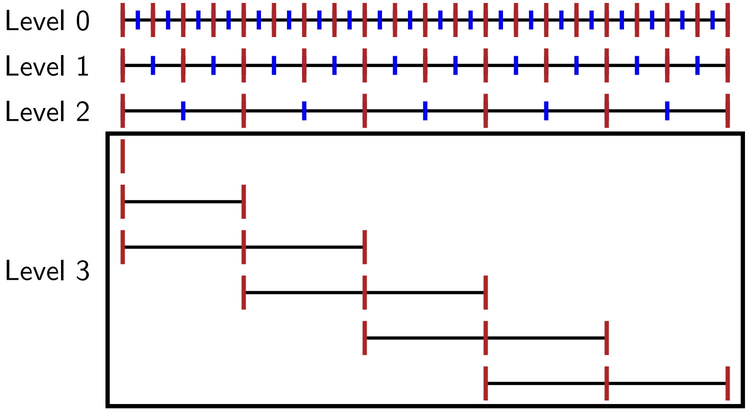


- Equivalent to MGRIT if $k = \#$points coarsest level
- Coarsest level structure can be used for any cycle types

# Numerical model

- Model "im_3_kw"* of an electrical machine
- Four-pole 3kW squirrel cage induction machine
- about 4,500 spatial DoFs

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* J. Gyselinck et. al. *Multi-slice FE modeling of electrical machines with skewed slots-the skew discretization error*, IEEE Magnetics **37**, 2001.

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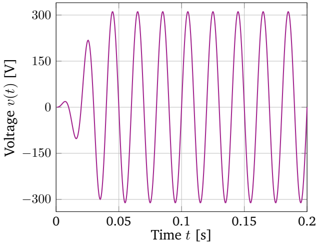


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- Excited with three-phase sinusoidal voltage of 50Hz and amplitude $\widehat{U} = 311.1$ V
- time interval: $[0, 0.2]$, 16,385 time points $(\Delta t \approx 10^{-5})$

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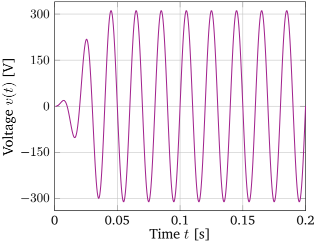
# Numerical model

**Optimization parameters:**
height and width

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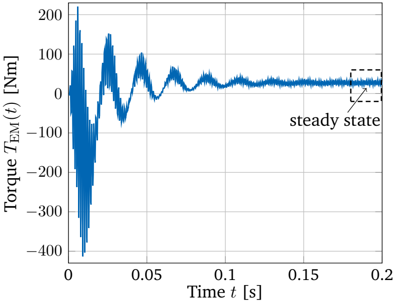
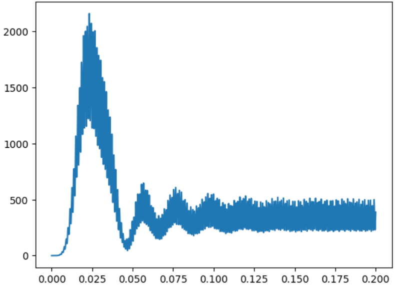
# Objective function




Nonlinear model "im_3_kw";
torque $T_{\mathsf{EM}} \rightsquigarrow P_{\mathsf{mech}} = T_{\mathsf{EM}}\omega_{\mathsf{mech}}$




Nonlinear model "im_3_kw";
Joule losses $P_{\mathsf{loss}}(\mathbf{A_3}, p)$



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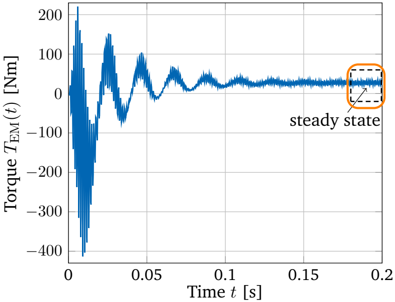


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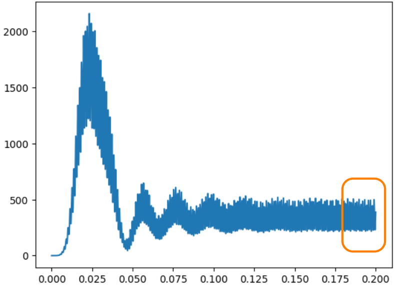



Nonlinear model "im_3_kw";
Joule losses $P_{\text{loss}}(\mathbf{A_3}, p)$

$$\min_{\mathbf{A_3},p} \qquad \widehat{J}(p) := -\frac{P_{\text{out}}(\mathbf{A_3}(p), p)}{P_{\text{in}}(\mathbf{A_3}, p)}$$

$$\text{s.t.} \qquad 0.007 \leq h \leq 0.015, \quad 0.0015 \leq w \leq 0.0035$$

$$P_{\text{out}}(\mathbf{A_3}, p) = \int_{0.18}^{0.2} P_{\text{mech}}(\mathbf{A_3}, p)\,\mathrm{d}t, \quad P_{\text{in}}(\mathbf{A_3}, p) = \int_{0.18}^{0.2} \left[P_{\text{mech}}(\mathbf{A_3}, p) + P_{\text{loss}}(\mathbf{A_3}, p)\right]\,\mathrm{d}t$$

# Optimization algorithm and software

## Optimization using Py-BOBYQA*

- Trust-region optimization algorithm BOBYQA (derivative-free)
- **Idea:** Use a model for the objective function (quadratic interpolation polynomial)

$$\mathcal{Q}^{(k)}(s) \approx \widehat{J}(p^{(k)} + s)$$

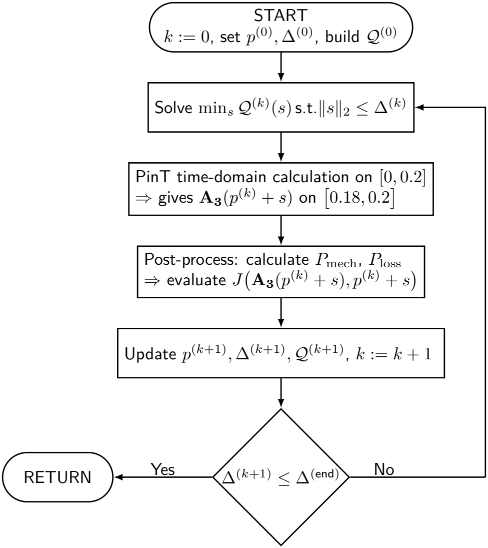
- Improve the model in every iteration:
  minimize $\mathcal{Q}^{(k)}$ inside a trust-region $\{s \in \mathbb{R}^n \ : \ \|s\|_2 \leq \Delta^{(k)}\}$

$$\min_s \mathcal{Q}^{(k)}(s) \text{ s. t. } \|s\|_2 \leq \Delta^{(k)}$$

- Initial trust-region radius: $\Delta^{(0)} = 10^{-4}$
- Stopping criterion: allowed trust-region radius $\Delta^{(\text{end})} = 10^{-8}$

*C. Cartis et. al., *Improving the flexibility and robustness of model-based derivative-free optimization solvers*, tech. report, University of Oxford, 2018.

# Optimization procedure



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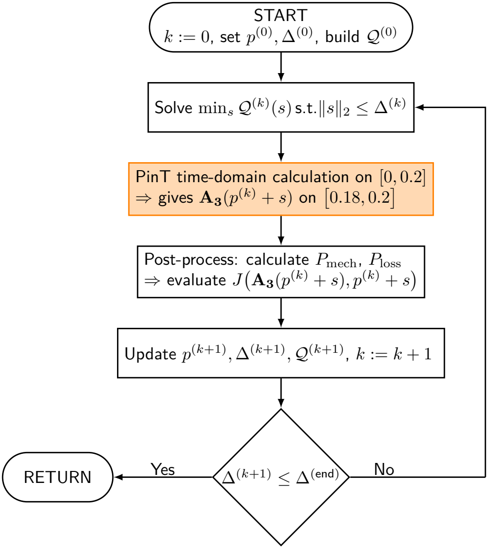
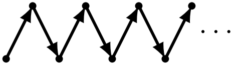


# PinT time-domain simulation and software

For each objective function evaluation, generate mesh using `Gmsh`

Time-domain simulation using PyMGRIT[a]

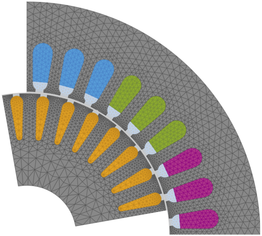
- Two-level AT-MGRIT with F-relaxation
- $m = 64$, local grid size: $k = 100$
- Initial guess: Full coarse-grid solve
- Subcycling on coarse level
- Spatial solves using GetDP[b]
- Stopping criterion: relative difference of Jule losses of two succ. iterations $< 1\%$
- 256 processes



[a] J. Hahne et. al., *PyMGRIT: A Python package for the parallel-in-time method MGRIT*, tech. report, University of Wuppertal, 2020.

[b] C. Geuzaine, *GetDP: a general finite-element solver for the de Rham complex, PAMM **7** (1), 2007.*

# Optimization results



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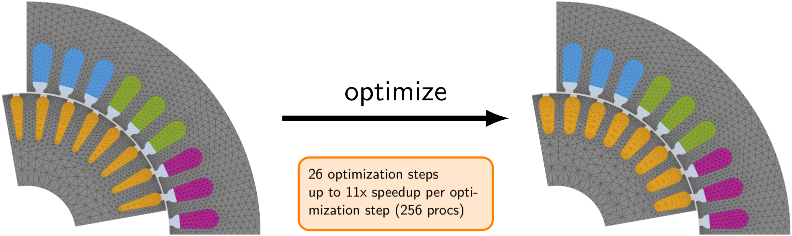


26 optimization steps
up to 11x speedup per optimization step (256 procs)

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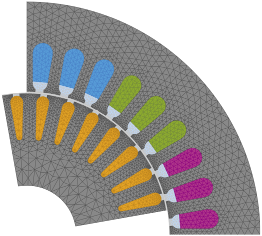
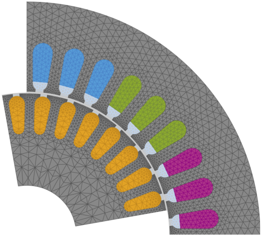


optimize



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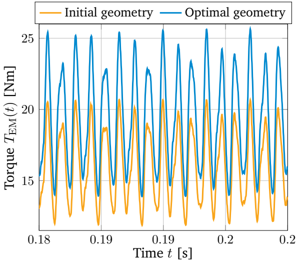
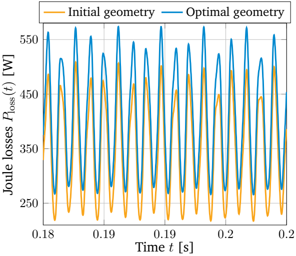
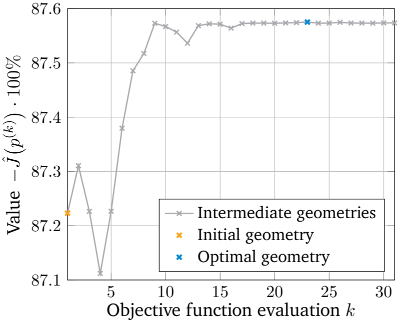


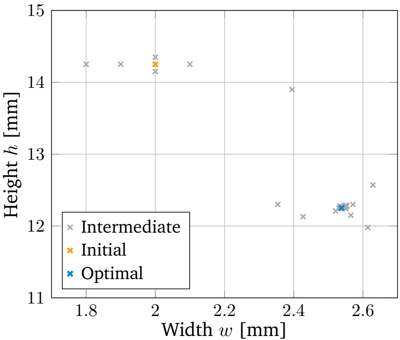

average increase of 16%




average increase of 20%

# Optimization process




31 function evaluations (optimal geometry found in iteration 18)

- one evaluation in each of the 26 iterations
- five additional evaluations for initial quadratic interpolation model

# Conclusions

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- Successfully applied AT-MGRIT in the design optimization of a realistic induction machine model.

## Software

- Code available in PyMGRIT (https://github.com/pymgrit/pymgrit)

Thank you

http:/timex-eurohpc.eu
http://parallel-in-time.org