Task graph-based performance analysis of PinT methods

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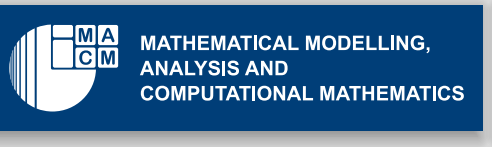

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- Challenges:
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- Approach:
  - Task-graph based on data-driven formulation of algorithms
  - Allows to cover any implementation using schedules of the tasks
  - Well known tool for load balancing techniques
  - Theoretical lower runtime bound based entirely on the algorithm

Parareal

- Decompose $[0, T]$ into N non-overlapping intervals

$$0 = T_0 < T_1 < ... < T_N = T$$

- Two propagation operators:
  - $\mathcal{F}$ expensive and high accurate
  - $\mathcal{G}$ cheap but less accurate
- Initalization:

$$\mathbf{u}_n^0 = \mathcal{G}(T_n, T_{n-1}, \mathbf{u}_{n-1}^0) \text{ for } n = 1, ..., N,$$

- Parareal iteration:

$$\mathbf{u}_n^{k+1} = \mathcal{F}(T_n, T_{n-1}, \mathbf{u}_{n-1}^k) + \mathcal{G}(T_n, T_{n-1}, \mathbf{u}_{n-1}^{k+1}) - \mathcal{G}(T_n, T_{n-1}, \mathbf{u}_{n-1}^k)$$

for $n = 1, ..., N$ and $k = 0, ...$

Parareal - data driven formulation

Algorithm: Parareal

$\mathbf{u}_0^0 \leftarrow \mathbf{u}_0$
for $i \leftarrow 1$ **to** N ▷ Compute initial guess
  $\tilde{\mathbf{u}}_i^0 \leftarrow \mathcal{G}(T_i, T_{i-1}, \mathbf{u}_{i-1}^0)$
  $\mathbf{u}_i^0 \leftarrow \tilde{\mathbf{u}}_i^0$
for $k \leftarrow 1$ **to** N ▷ Parareal iterations
  foreach $i \in \{k, k+1, \ldots, N\}$ **do**
    $\hat{\mathbf{u}}_i^{k-1} \leftarrow \mathcal{F}(T_i, T_{i-1}, \mathbf{u}_{i-1}^{k-1})$
  for $i \leftarrow k$ **to** N
    $\tilde{\mathbf{u}}_i^k \leftarrow \mathcal{G}(T_i, T_{i-1}, \mathbf{u}_{i-1}^{\min(k,i-1)})$
    $\mathbf{u}_i^k \leftarrow \tilde{\mathbf{u}}_i^k + \hat{\mathbf{u}}_i^{k-1} - \tilde{\mathbf{u}}_i^{k-1}$

- Typically only $K \ll N$ iterations required

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  - Deferred correction approach instead of fine and coarse propagator
  - Arbitrary hierarchy of levels
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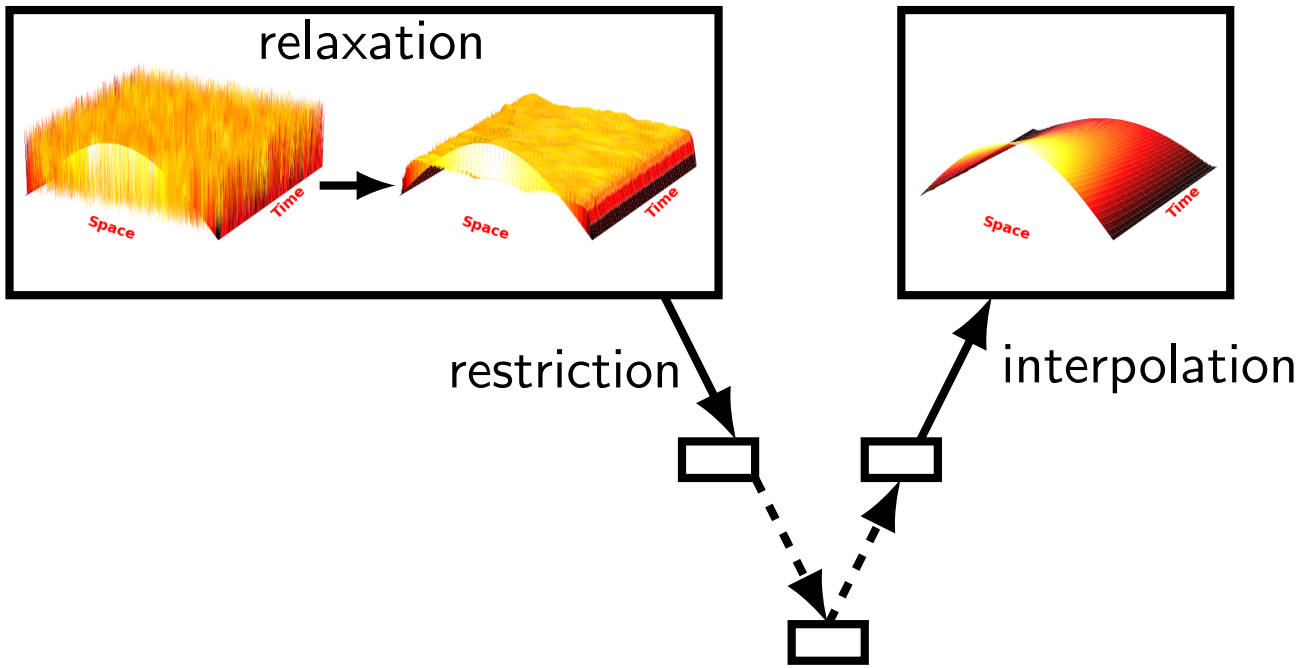
- Spectral deferred correction methods
  - Pipelined version of multi-level SDC with each time slice performing SDC sweeps in parallel

Multigrid-reduction-in-time (MGRIT)



- Reduction-based time-multigrid method
- In specialized two-level setting: MGRIT $\equiv$ Parareal

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  - Local: Measurement of solution quality at individual points in time
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- Both can be formulated in a data-driven formulation

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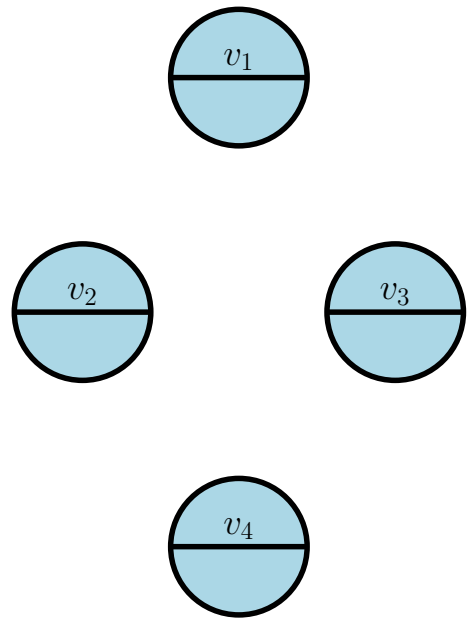

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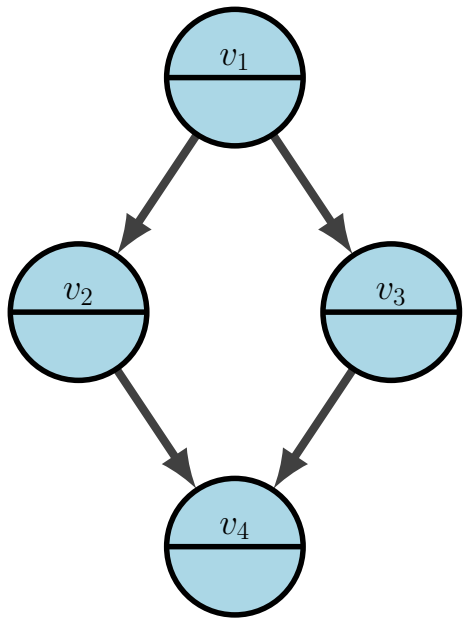

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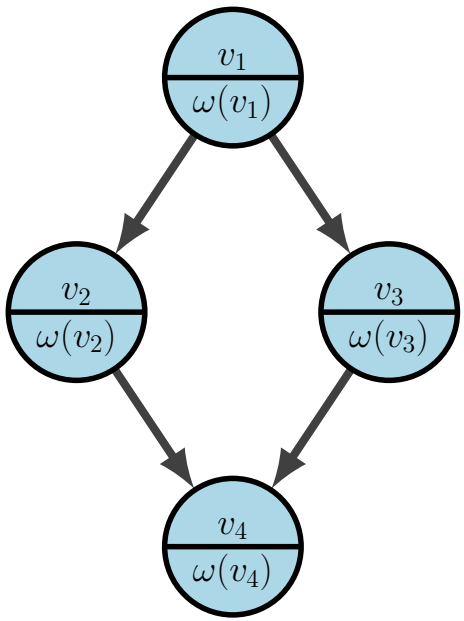

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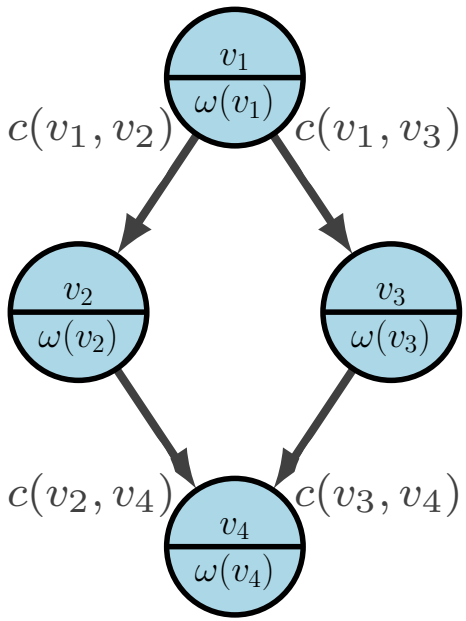
- Edge weights $c : E \to \mathbb{R}_0^+$
  - Represent communication cost between tasks



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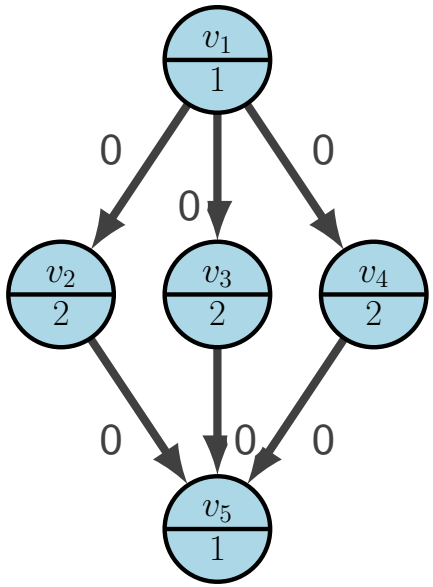
- If $c(e) = 0 \; \forall e \in E$: Minimum possible makespan for $N_P = \infty$ is longest path within the graph

Schedule example



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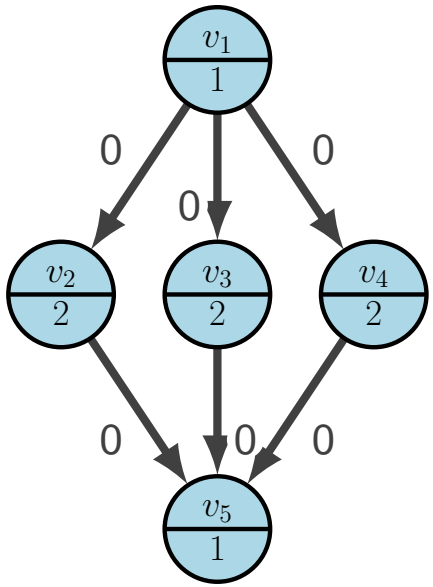

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A(v)
$v_1 \rightarrow p_1$
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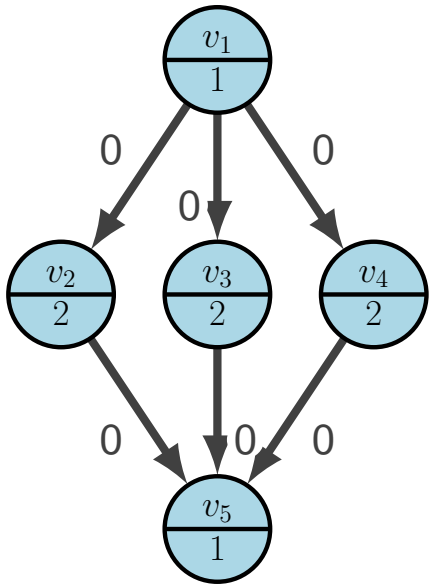

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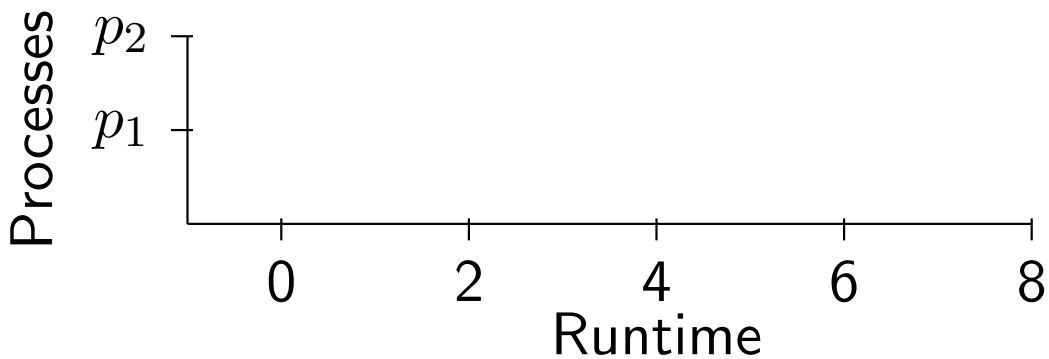

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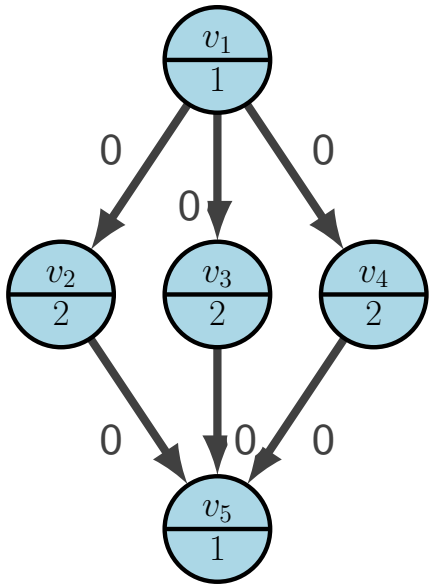

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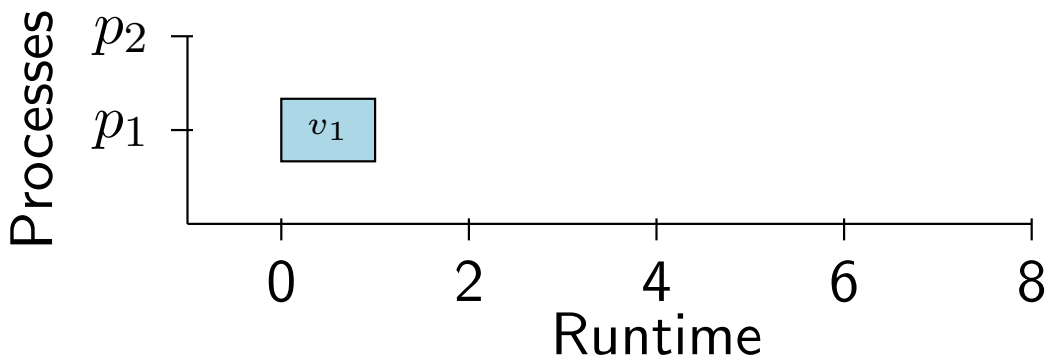

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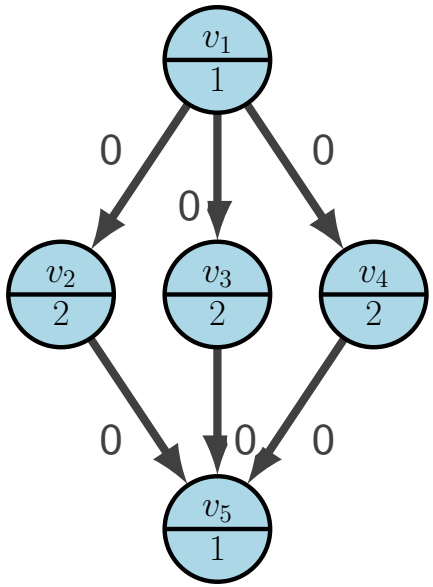

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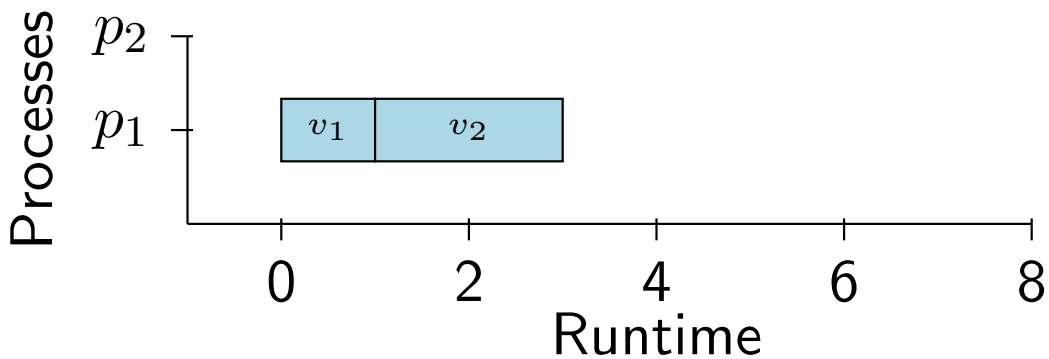

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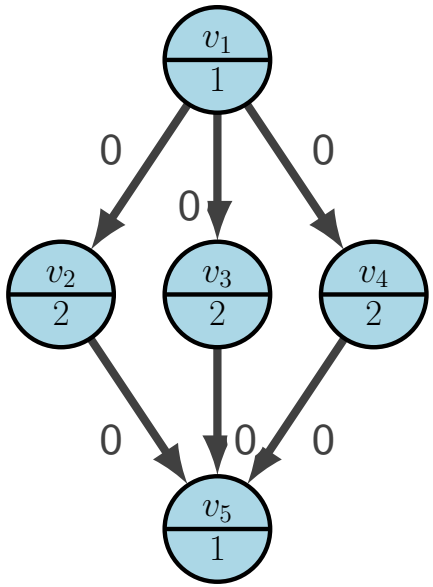

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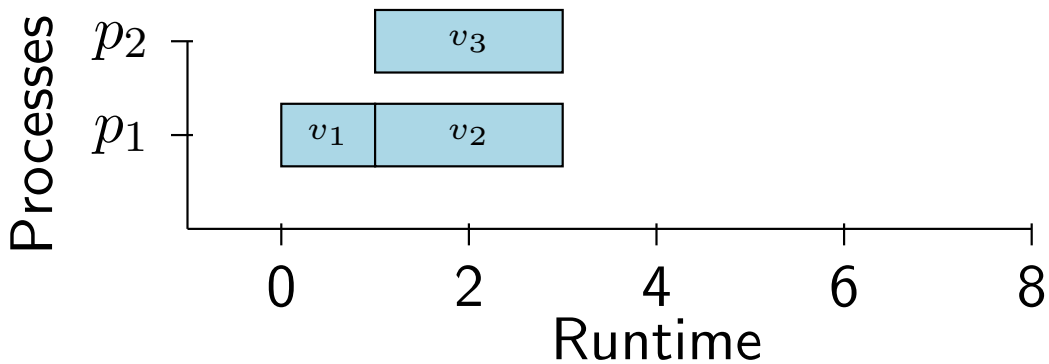

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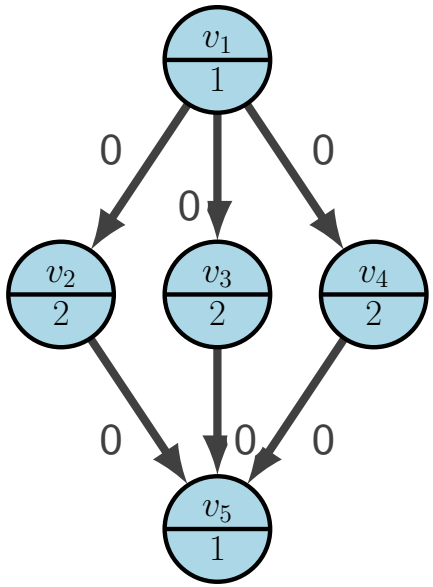

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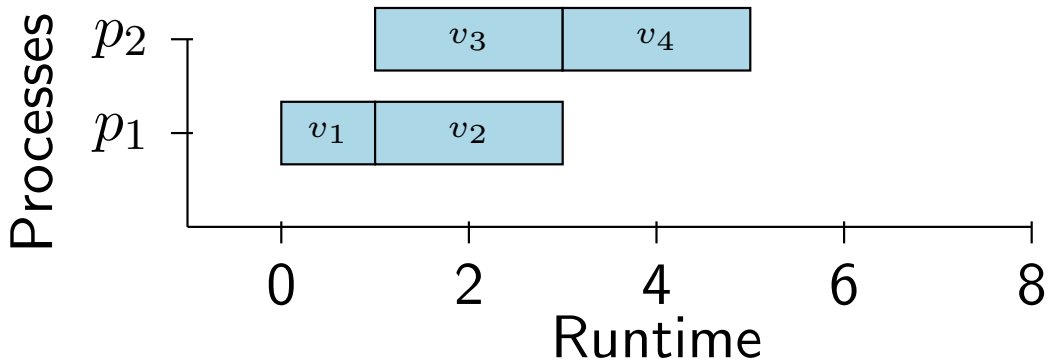

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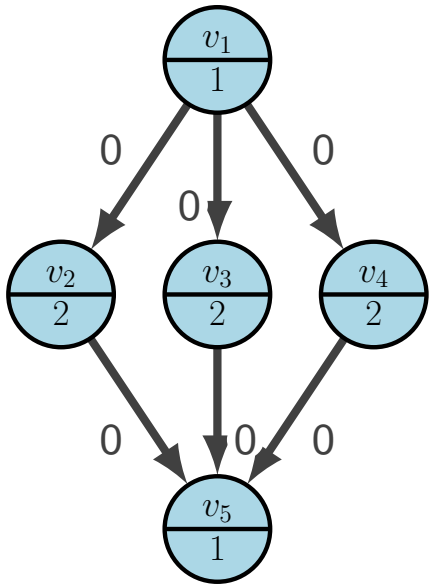

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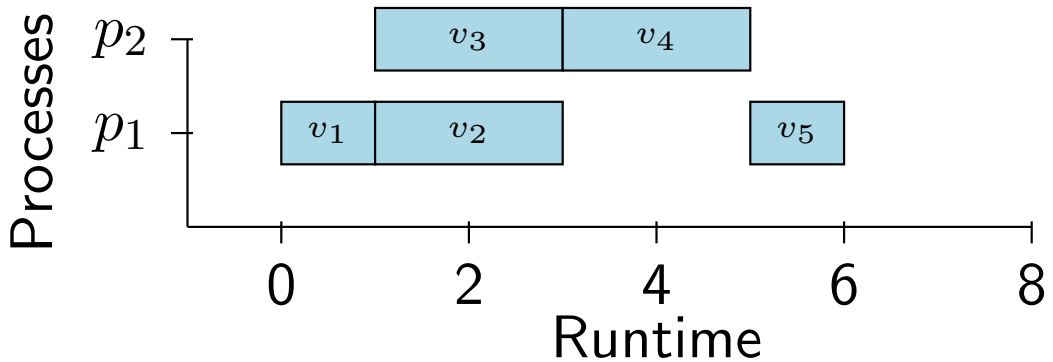

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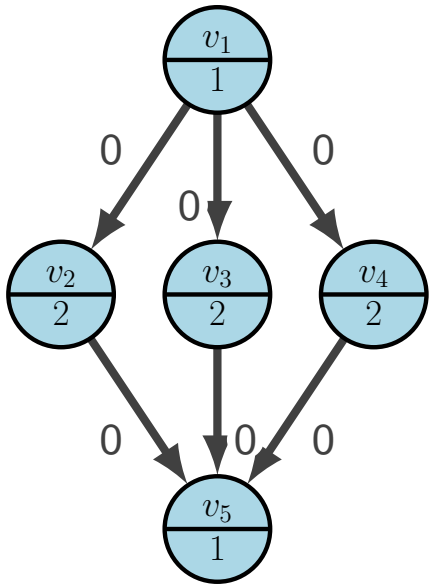

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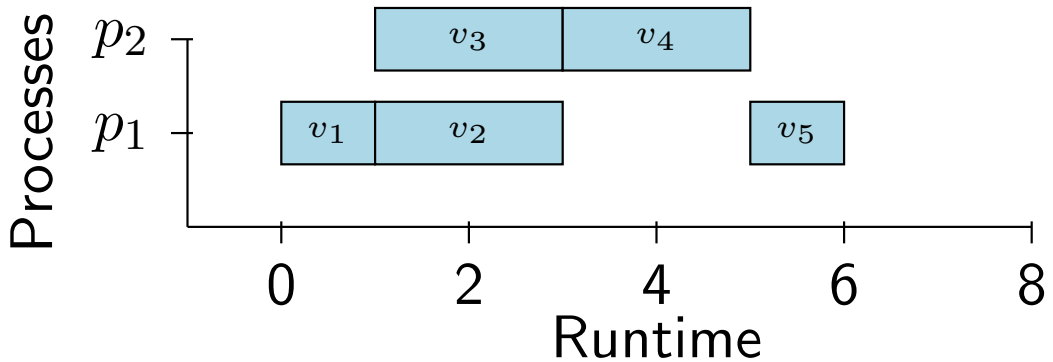

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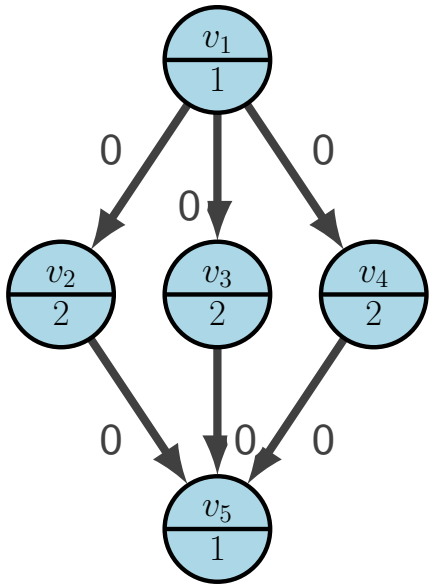

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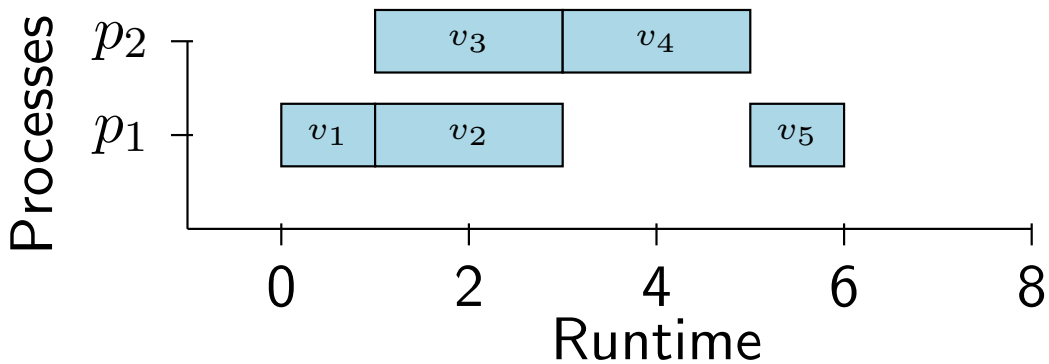

- Makespan of schedule: 6
- Longest path $\{v_1 \rightarrow v_2 \rightarrow v_5\} \Rightarrow$ minimal parallel runtime: 4

Task graph for Parareal $N = 3$

Algorithm: Parareal

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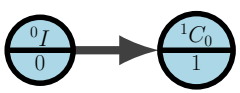

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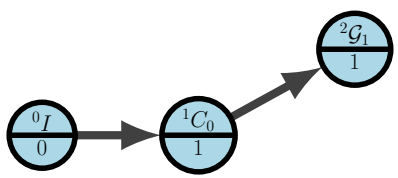

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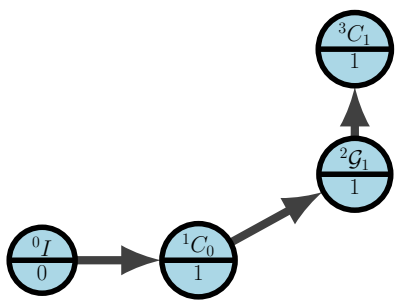

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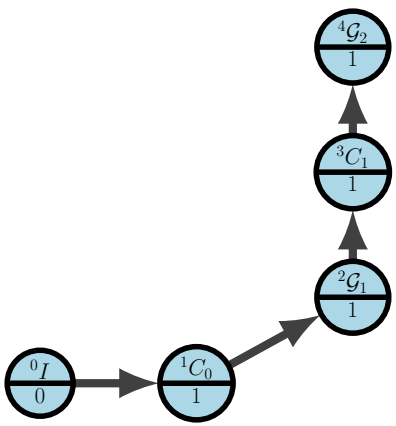

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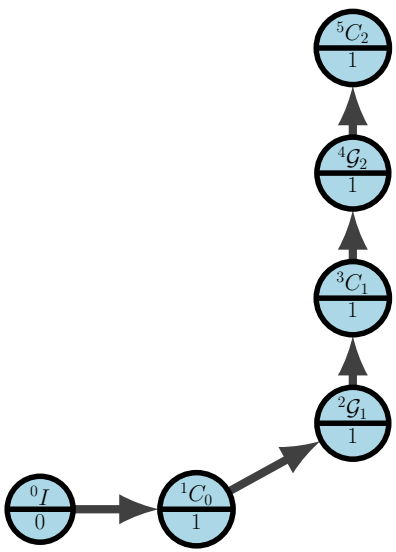

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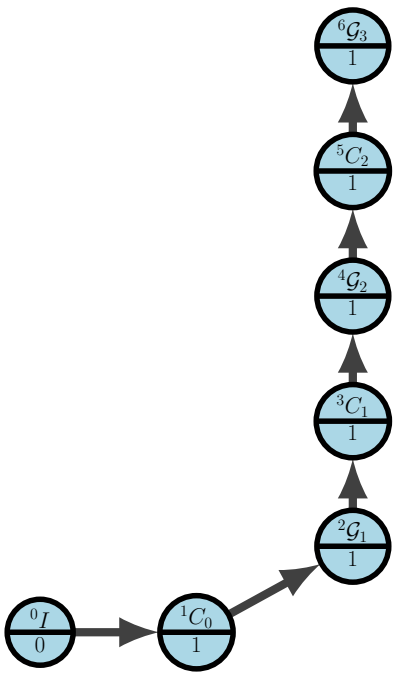

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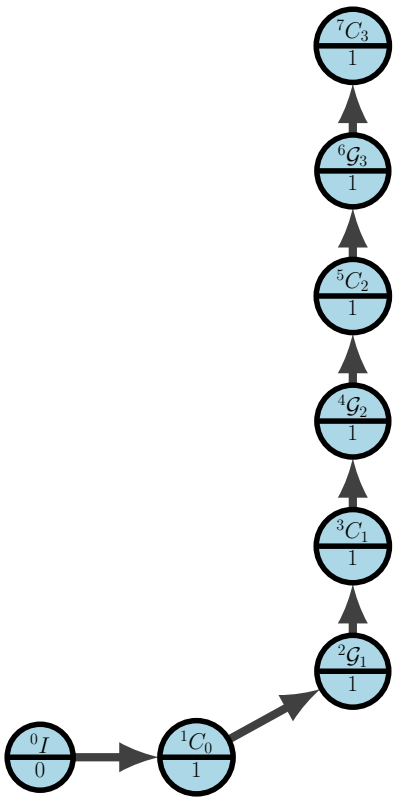

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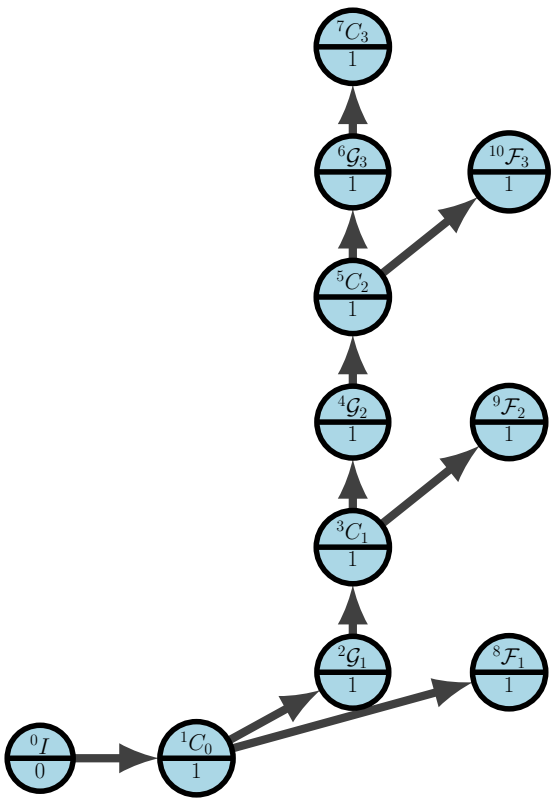

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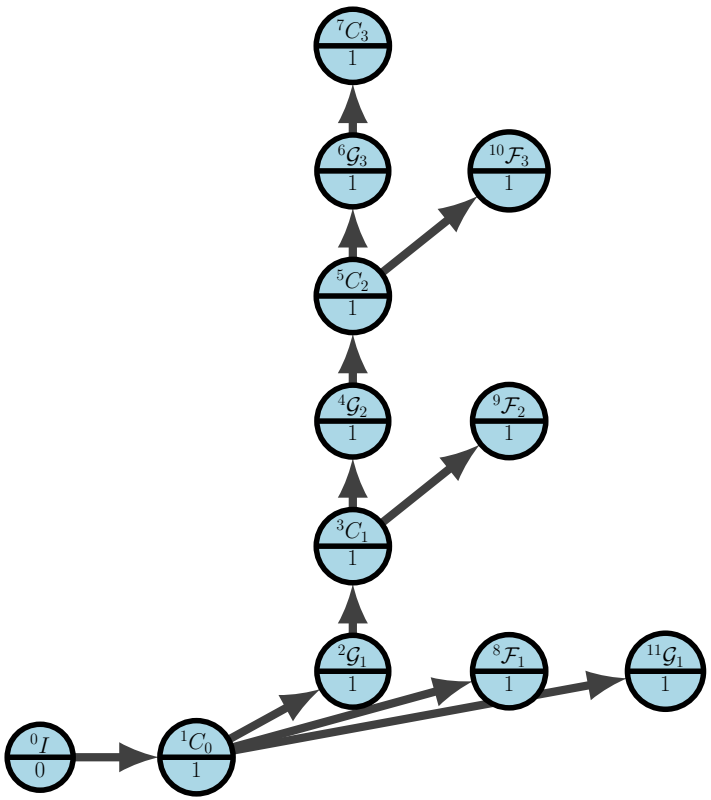

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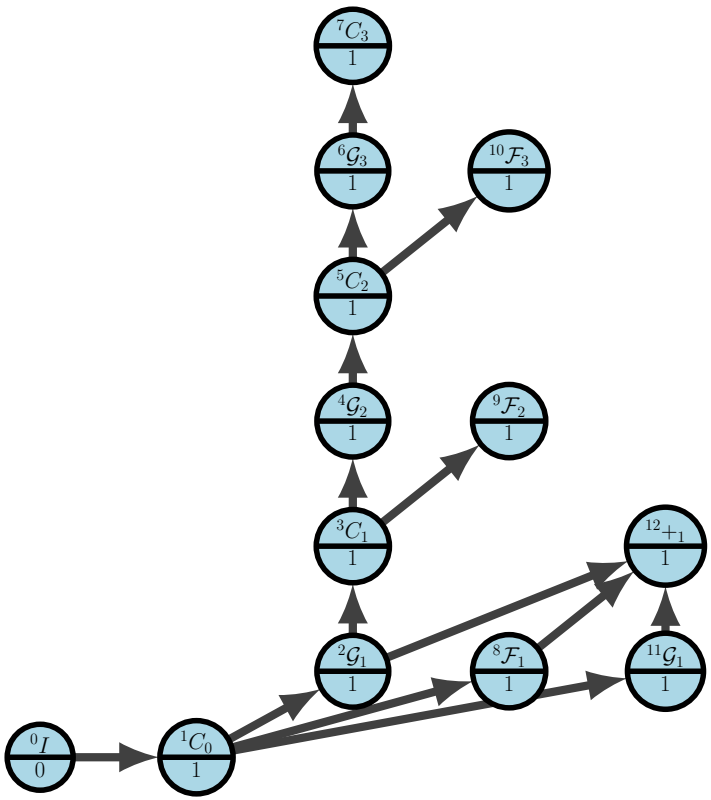

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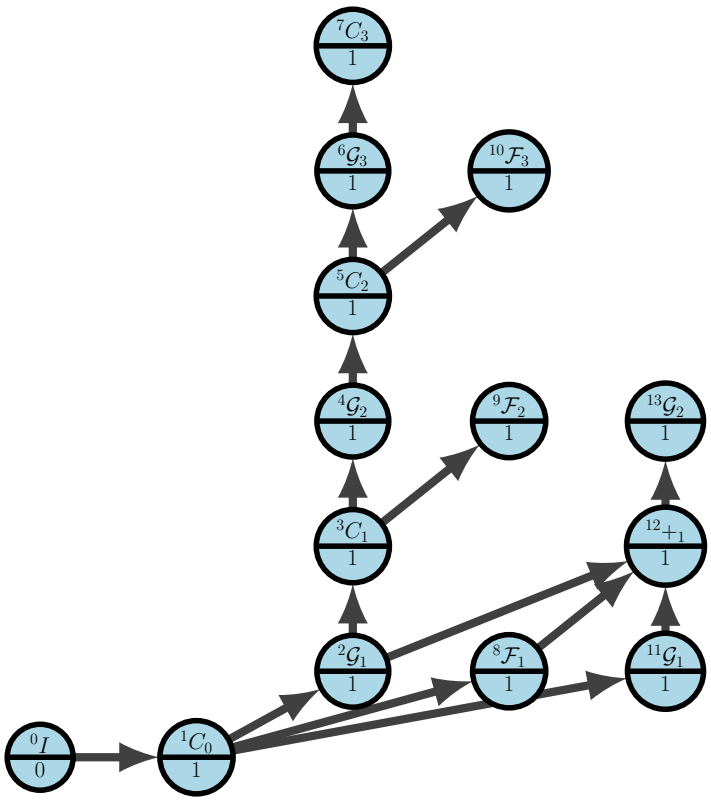

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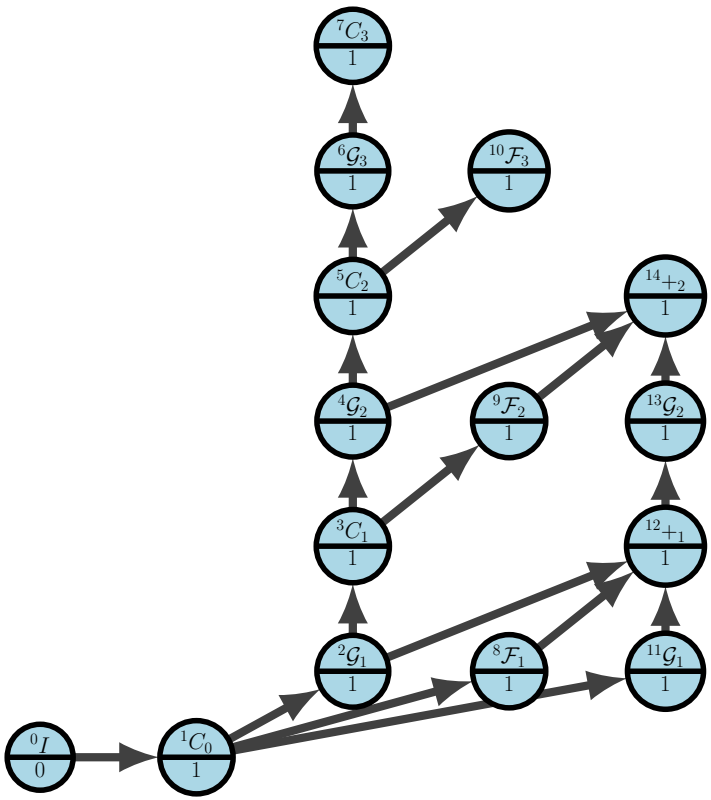

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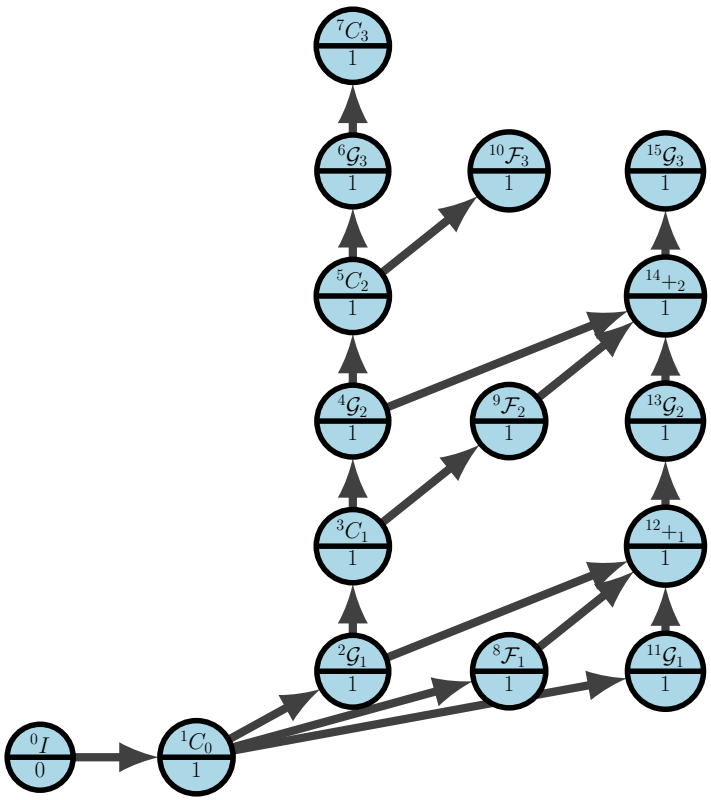

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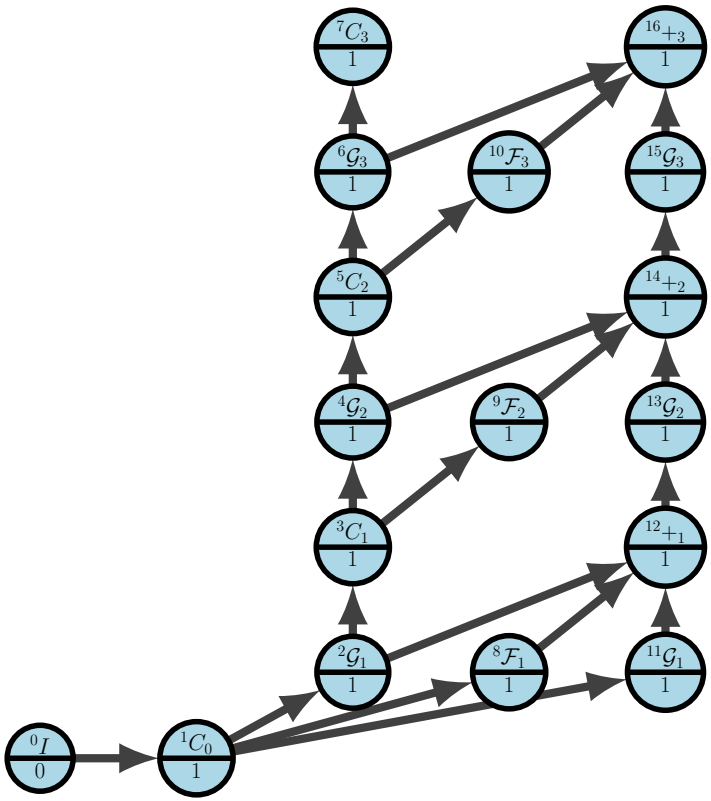

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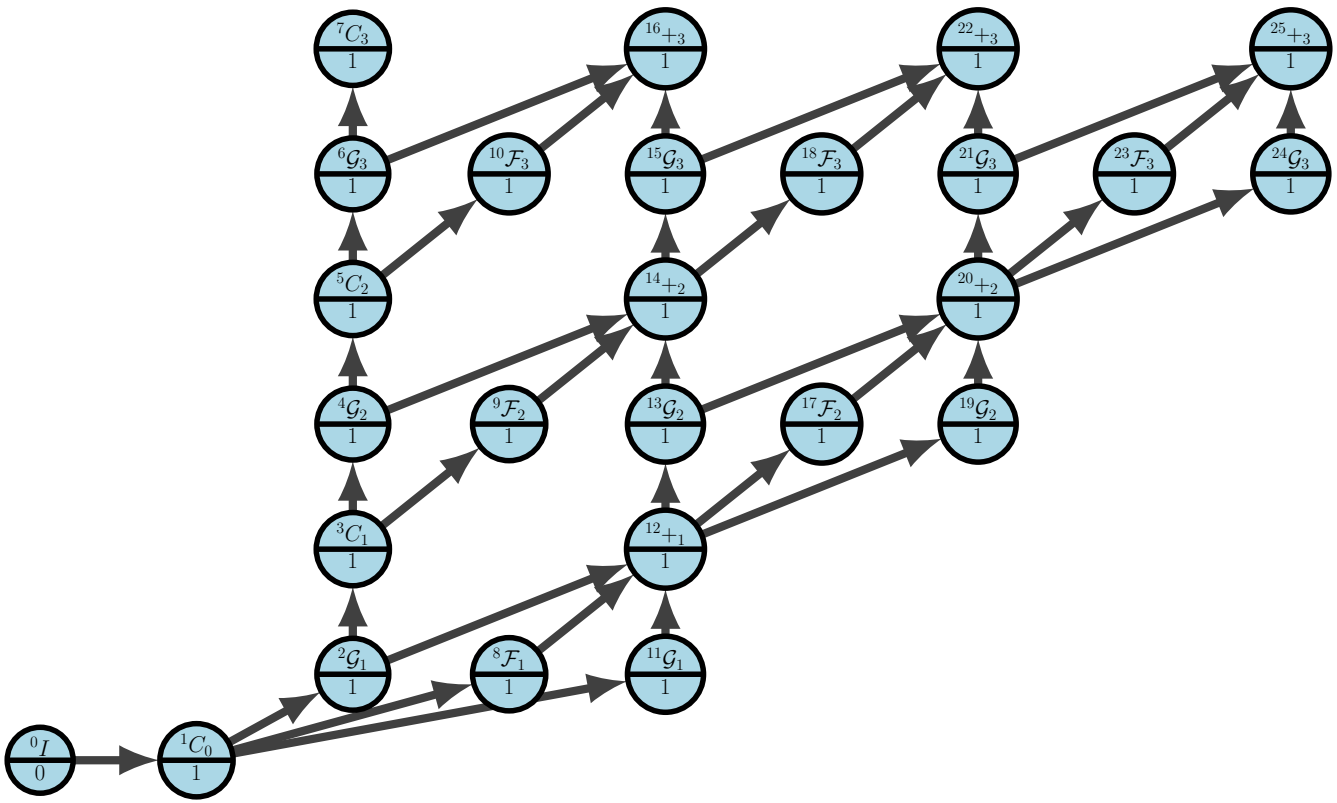

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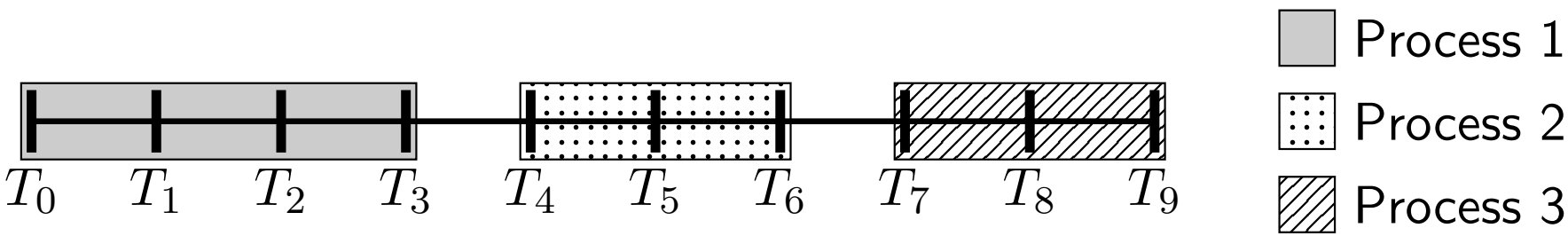

PinT parallelization strategies

- Typically static allocation of time points to processes
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- Two types of allocation:
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  - Windowing with block-by-block (applying method multiple times)

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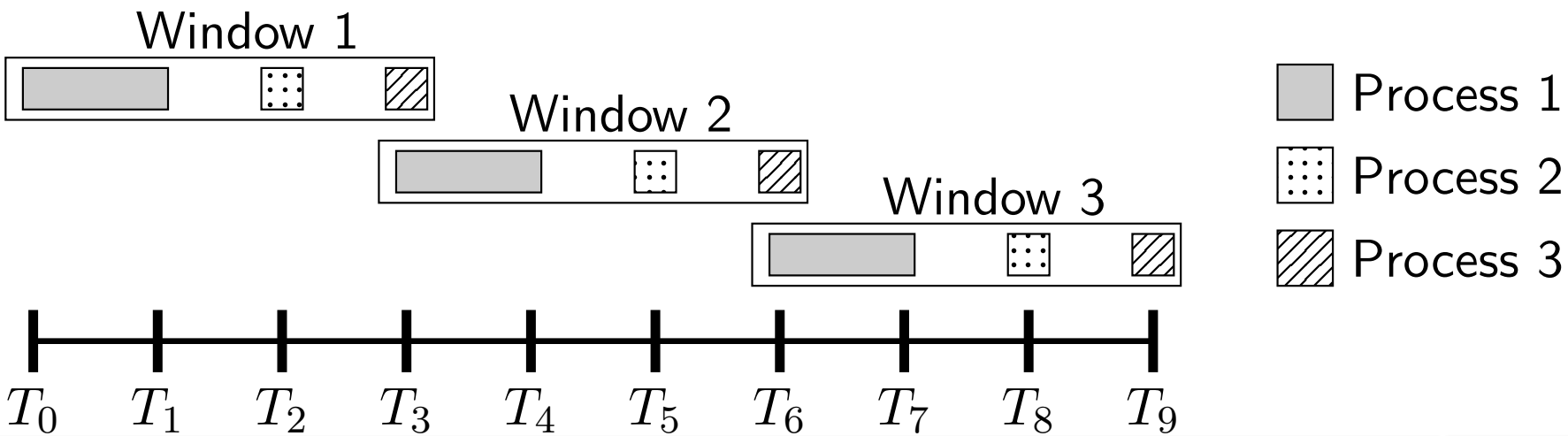
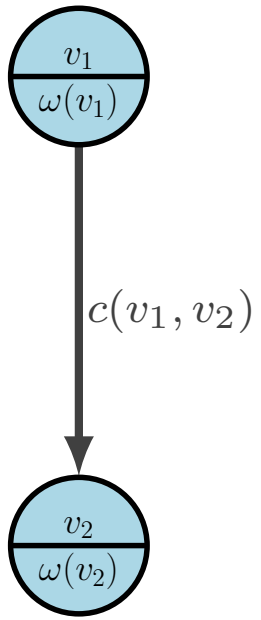


Weighting of tasks and communication



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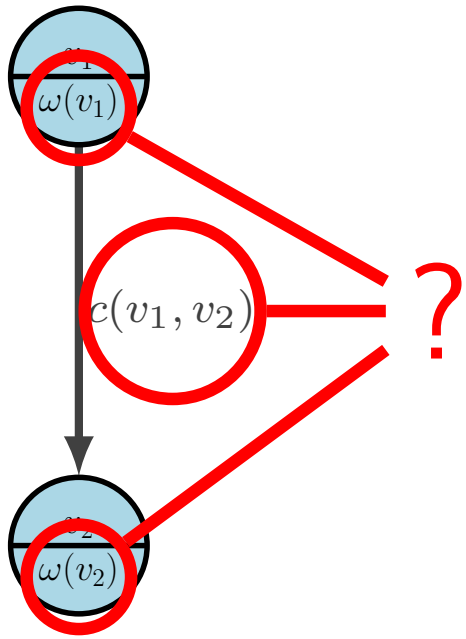

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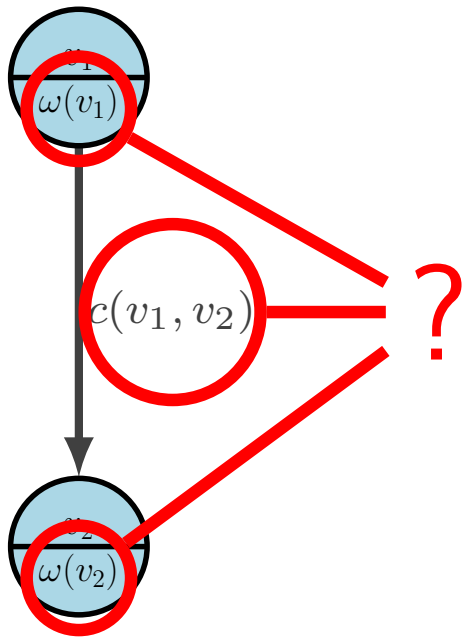

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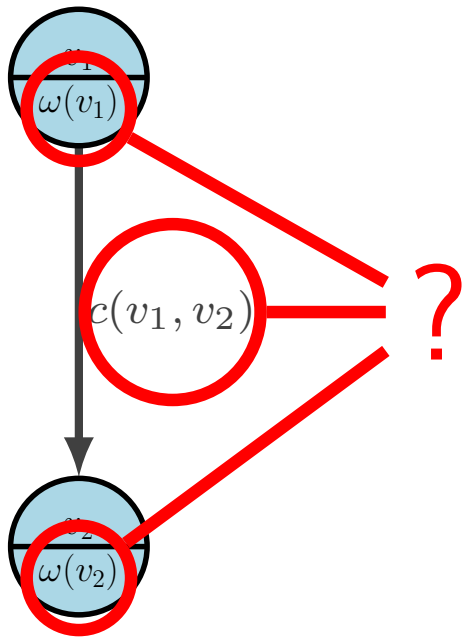
- Decision depends on use case



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- Pseudo spatial problem:
  - Each required function sleeps only for a controllable time
  - Enables large test sets in terms of task costs

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PFASST - setting

- LibPFASST
  - Classical view
  - Burn-in prediction phase
- PySDC
  - Multigrid view
  - Fine-sweep prediction phase
- Both:
  - Local convergence criterion
  - Implicit sweeper (pre-implemented)
  - Cost per implicit solve: 0.05 s
  - All other costs: 0 s
  - Runtime of operations is measured (reusable for other problems)
  - Communication cost: 0

Parameters	
Number of time intervals	N
Number processes time dimension	N_P
Number of levels	L
Number of iterations	K
Convergence criterion	
Two "views"	
Number of sweeps on level ℓ	
Collocation nodes on level ℓ	
Predictor type	
Skip fine-level sweep at start/end	
Runtime communication in time	
Runtime convergence criterion	
Runtime `SDCSweep` at level ℓ	
Runtime `FEvalAll` at level ℓ	
Runtime `FEvalSingle` at level ℓ	
Runtime `RestrictAll` at level ℓ	
Runtime `RestrictSingle` at level ℓ	
Runtime `InterpolateAll` at level ℓ	
Runtime `InterpolateSingle` at level ℓ	
Runtime `FAS` at level ℓ	

LibPFASST - model vs. runtime

N	N_P	L	K	# Sweeps	collocation nodes	`LibPFASST` runtime	Model	L. b. ($N_P = \infty$)
16	16	2	10	$(1, 1)$	$(5, 3)$	5.06	5.05	5.04
16	16	2	10	$(1, 1)$	$(7, 5)$	8.88	8.87	8.86
16	16	2	10	$(2, 1)$	$(5, 3)$	7.07	7.04	7.03
16	16	2	10	$(2, 1)$	$(7, 5)$	11.9	11.89	11.88
32	32	4	10	$(1, 1, 1, 1)$	$(7, 5, 3, 2)$	12.16	12.12	12.11
32	32	4	10	$(2, 1, 1, 1)$	$(7, 5, 3, 2)$	15.17	15.13	15.12
32	32	4	10	$(1, 2, 1, 1)$	$(7, 5, 3, 2)$	16.17	16.14	16.13
32	32	4	10	$(1, 1, 2, 1)$	$(7, 5, 3, 2)$	14.17	14.14	14.13
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- Actual runtime very close to predictions
- Lower bound (L.b.) and predictions very similar

PySDC - model vs. runtime

N	N_P	L	K	# Sweeps	collocation nodes	pySDC runtime	Model	L. b. ($N_P = \infty$)
16	16	2	10	$(1, 1)$	$(5, 3)$	5.22	5.00	5.00
16	16	2	10	$(1, 1)$	$(7, 5)$	9.45	8.99	8.99
16	16	2	10	$(2, 1)$	$(5, 3)$	7.38	7.07	7.07
16	16	2	10	$(2, 1)$	$(7, 5)$	12.78	12.23	12.23
32	32	4	10	$(1, 1, 1, 1)$	$(7, 5, 3, 2)$	12.82	12.34	12.34
32	32	4	10	$(2, 1, 1, 1)$	$(7, 5, 3, 2)$	16.23	15.57	15.56
32	32	4	10	$(1, 2, 1, 1)$	$(7, 5, 3, 2)$	17.16	16.41	16.30
32	32	4	10	$(1, 1, 2, 1)$	$(7, 5, 3, 2)$	14.96	14.26	13.22
32	32	4	10	$(1, 2, 2, 1)$	$(7, 5, 3, 2)$	19.38	18.57	15.43

PySDC - model vs. runtime

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16	16	2	10	$(1, 1)$	$(5, 3)$	5.22	5.00	5.00
16	16	2	10	$(1, 1)$	$(7, 5)$	9.45	8.99	8.99
16	16	2	10	$(2, 1)$	$(5, 3)$	7.38	7.07	7.07
16	16	2	10	$(2, 1)$	$(7, 5)$	12.78	12.23	12.23
32	32	4	10	$(1, 1, 1, 1)$	$(7, 5, 3, 2)$	12.82	12.34	12.34
32	32	4	10	$(2, 1, 1, 1)$	$(7, 5, 3, 2)$	16.23	15.57	15.56
32	32	4	10	$(1, 2, 1, 1)$	$(7, 5, 3, 2)$	17.16	16.41	16.30
32	32	4	10	$(1, 1, 2, 1)$	$(7, 5, 3, 2)$	14.96	14.26	13.22
32	32	4	10	$(1, 2, 2, 1)$	$(7, 5, 3, 2)$	19.38	18.57	15.43

- Actual runtime differs by no more than 5%

PySDC - model vs. runtime

N	N_P	L	K	# Sweeps	collocation nodes	pySDC runtime	Model	L. b. ($N_P = \infty$)
16	16	2	10	$(1, 1)$	$(5, 3)$	5.22	5.00	5.00
16	16	2	10	$(1, 1)$	$(7, 5)$	9.45	8.99	8.99
16	16	2	10	$(2, 1)$	$(5, 3)$	7.38	7.07	7.07
16	16	2	10	$(2, 1)$	$(7, 5)$	12.78	12.23	12.23
32	32	4	10	$(1, 1, 1, 1)$	$(7, 5, 3, 2)$	12.82	12.34	12.34
32	32	4	10	$(2, 1, 1, 1)$	$(7, 5, 3, 2)$	16.23	15.57	15.56
32	32	4	10	$(1, 2, 1, 1)$	$(7, 5, 3, 2)$	17.16	16.41	16.30
32	32	4	10	$(1, 1, 2, 1)$	$(7, 5, 3, 2)$	14.96	14.26	13.22
32	32	4	10	$(1, 2, 2, 1)$	$(7, 5, 3, 2)$	19.38	18.57	15.43

- Actual runtime differs by no more than 5%
- Lower bound and predictions similiar for most settings

PySDC - model vs. runtime

N	N_P	L	K	# Sweeps	collocation nodes	pySDC runtime	Model	L. b. ($N_P = \infty$)
16	16	2	10	$(1, 1)$	$(5, 3)$	5.22	5.00	5.00
16	16	2	10	$(1, 1)$	$(7, 5)$	9.45	8.99	8.99
16	16	2	10	$(2, 1)$	$(5, 3)$	7.38	7.07	7.07
16	16	2	10	$(2, 1)$	$(7, 5)$	12.78	12.23	12.23
32	32	4	10	$(1, 1, 1, 1)$	$(7, 5, 3, 2)$	12.82	12.34	12.34
32	32	4	10	$(2, 1, 1, 1)$	$(7, 5, 3, 2)$	16.23	15.57	15.56
32	32	4	10	$(1, 2, 1, 1)$	$(7, 5, 3, 2)$	17.16	16.41	16.30
32	32	4	10	$(1, 1, 2, 1)$	$(7, 5, 3, 2)$	14.96	14.26	13.22
32	32	4	10	$(1, 2, 2, 1)$	$(7, 5, 3, 2)$	19.38	18.57	15.43

- Actual runtime differs by no more than 5%
- Lower bound and predictions similiar for most settings
- Small derivations for some four-level settings (PFASST multigrid view)

MGRIT - setting

- XBRAID
  - Global convergence criterion
- PyMGRIT
  - Local and global convergence criteria
- Both:
  - Cost per time integration: 0.05 s
  - All other costs: 0 s
  - Runtime of operations is measured (reusable for other problems)
  - Communication cost: 0

Parameters	
Number of time intervals	N
Number processes time dimension	N_P
Number of levels	L
Number of iterations	K
Convergence criterion	
Coarsening factor from level ℓ to level $\ell + 1$	
Cycle type	
Nested iterations	
Skip down	
Number of CF-relaxations on level ℓ	
Runtime communication in time	
Runtime convergence criterion	
Runtime time integrator at level ℓ	
Runtime spatial restriction at level ℓ	
Runtime spatial interpolation at level ℓ	

XBRAID - model vs. runtime

N	N_P	L	K	cyc.	coarsening	$\#\ CF$-relax.	skip. down	XBRAID runtime	Model	L.b. ($N_P = \infty$)
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	True	19.02	18.93	15.61
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	False	22.79	21.83	17.91
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	True	21.59	21.51	13.63
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	False	23.90	23.01	14.99
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	True	22.81	22.66	11.88
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	False	26.38	26.16	13.43
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	True	30.71	30.64	15.17
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	False	32.61	32.49	16.04

XBRAID - model vs. runtime

N	N_P	L	K	cyc.	coarsening	# CF-relax.	skip. down	XBRAID runtime	Model	L.b. ($N_P = \infty$)
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	True	19.02	18.93	15.61
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	False	22.79	21.83	17.91
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	True	21.59	21.51	13.63
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	False	23.90	23.01	14.99
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	True	22.81	22.66	11.88
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	False	26.38	26.16	13.43
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	True	30.71	30.64	15.17
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	False	32.61	32.49	16.04

- Actual runtime very close to predictions

XBRAID - model vs. runtime

N	N_P	L	K	cyc.	coarsening	$\#\ CF$-relax.	skip. down	XBRAID runtime	Model	L.b. ($N_P = \infty$)
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	True	19.02	18.93	15.61
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	False	22.79	21.83	17.91
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	True	21.59	21.51	13.63
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	False	23.90	23.01	14.99
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	True	22.81	22.66	11.88
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	False	26.38	26.16	13.43
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	True	30.71	30.64	15.17
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	False	32.61	32.49	16.04

- Actual runtime very close to predictions
  - Cluster based problem where the first communication between nodes is especially expensive

XBRAID - model vs. runtime

N	N_P	L	K	cyc.	coarsening	$\#\ CF$-relax.	skip. down	XBRAID runtime	Model	L.b. ($N_P = \infty$)
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	True	19.02	18.93	15.61
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	False	22.79	21.83	17.91
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	True	21.59	21.51	13.63
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	False	23.90	23.01	14.99
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	True	22.81	22.66	11.88
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	False	26.38	26.16	13.43
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	True	30.71	30.64	15.17
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	False	32.61	32.49	16.04

- Actual runtime very close to predictions
  - Cluster based problem where the first communication between nodes is especially expensive
- Theoretically further potential
  - Largely due to global convergence criterion (blocking communication)

XBRAID - model vs. runtime

N	N_P	L	K	cyc.	coarsening	$\#\ CF$-relax.	skip. down	XBRAID runtime	Model	L.b. ($N_P = \infty$)
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	True	19.02	18.93	15.61
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	False	22.79	21.83	17.91
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	True	21.59	21.51	13.63
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	False	23.90	23.01	14.99
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	True	22.81	22.66	11.88
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	False	26.38	26.16	13.43
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	True	30.71	30.64	15.17
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	False	32.61	32.49	16.04

- Actual runtime very close to predictions
  - Cluster based problem where the first communication between nodes is especially expensive
- Theoretically further potential
  - Largely due to global convergence criterion (blocking communication)
  - Comparison between 256 and ∞ processes

PyMGRIT - model vs. runtime

N	N_P	L	K	cyc.	coarsening	# CF-relax.	nest. iter.	conv. crit.	PyMGRIT runtime	Model	L.b. ($N_P = \infty$)
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	T	local	19.99	19.53	18.01
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	F	local	18.87	18.43	18.02
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	T	local	17.69	17.17	15.09
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	F	local	16.98	16.66	15.09
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	T	global	24.66	23.91	18.06
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	F	global	23.28	21.94	18.01
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	T	global	25.08	24.04	15.07
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	F	global	24.24	23.15	15.08
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	T	local	25.82	25.36	13.52
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	T	local	26.06	25.43	16.13
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	T	global	30.91	29.97	13.49
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	T	global	35.93	34.82	16.14

PyMGRIT - model vs. runtime

N	N_P	L	K	cyc.	coarsening	# CF-relax.	nest. iter.	conv. crit.	PyMGRIT runtime	Model	L.b. ($N_P = \infty$)
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	T	local	19.99	19.53	18.01
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	F	local	18.87	18.43	18.02
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	T	local	17.69	17.17	15.09
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	F	local	16.98	16.66	15.09
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	T	global	24.66	23.91	18.06
4096	256	5	6	V	$(16, 4, 4, 4)$	$(1, 1, 1, 1)$	F	global	23.28	21.94	18.01
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	T	global	25.08	24.04	15.07
4096	256	5	6	F	$(16, 4, 4, 4)$	$(0, 0, 0, 0)$	F	global	24.24	23.15	15.08
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	T	local	25.82	25.36	13.52
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	T	local	26.06	25.43	16.13
4096	256	6	6	V	$(4, 4, 4, 4, 4)$	$(1, 1, 1, 1, 1)$	T	global	30.91	29.97	13.49
4096	256	6	6	F	$(4, 4, 4, 4, 4)$	$(0, 0, 0, 0, 0)$	T	global	35.93	34.82	16.14

- Actual runtime very close to predictions
  - Same cluster based problem

PyMGRIT - model vs. runtime

N	N_P	L	K	cyc.	coarsening	# CF-relax.	nest. iter.	conv. crit.	PyMGRIT runtime	Model	L.b. ($N_P = \infty$)
4096	256	5	6	V	$(16,4,4,4)$	$(1,1,1,1)$	T	local	19.99	19.53	18.01
4096	256	5	6	V	$(16,4,4,4)$	$(1,1,1,1)$	F	local	18.87	18.43	18.02
4096	256	5	6	F	$(16,4,4,4)$	$(0,0,0,0)$	T	local	17.69	17.17	15.09
4096	256	5	6	F	$(16,4,4,4)$	$(0,0,0,0)$	F	local	16.98	16.66	15.09
4096	256	5	6	V	$(16,4,4,4)$	$(1,1,1,1)$	T	global	24.66	23.91	18.06
4096	256	5	6	V	$(16,4,4,4)$	$(1,1,1,1)$	F	global	23.28	21.94	18.01
4096	256	5	6	F	$(16,4,4,4)$	$(0,0,0,0)$	T	global	25.08	24.04	15.07
4096	256	5	6	F	$(16,4,4,4)$	$(0,0,0,0)$	F	global	24.24	23.15	15.08
4096	256	6	6	V	$(4,4,4,4,4)$	$(1,1,1,1,1)$	T	local	25.82	25.36	13.52
4096	256	6	6	F	$(4,4,4,4,4)$	$(0,0,0,0,0)$	T	local	26.06	25.43	16.13
4096	256	6	6	V	$(4,4,4,4,4)$	$(1,1,1,1,1)$	T	global	30.91	29.97	13.49
4096	256	6	6	F	$(4,4,4,4,4)$	$(0,0,0,0,0)$	T	global	35.93	34.82	16.14

- Actual runtime very close to predictions
  - Same cluster based problem
- Model approaches the theoretical limit when local criterion is used

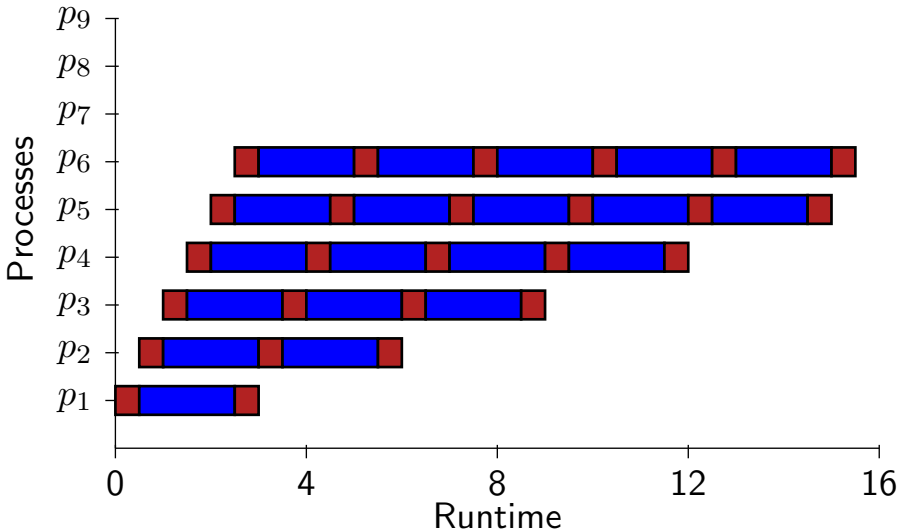
PyMGRIT - model vs. runtime

N	N_P	L	K	cyc.	coarsening	# CF-relax.	nest. iter.	conv. crit.	PyMGRIT runtime	Model	L.b. ($N_P = \infty$)
4096	256	5	6	V	$(16,4,4,4)$	$(1,1,1,1)$	T	local	19.99	19.53	18.01
4096	256	5	6	V	$(16,4,4,4)$	$(1,1,1,1)$	F	local	18.87	18.43	18.02
4096	256	5	6	F	$(16,4,4,4)$	$(0,0,0,0)$	T	local	17.69	17.17	15.09
4096	256	5	6	F	$(16,4,4,4)$	$(0,0,0,0)$	F	local	16.98	16.66	15.09
4096	256	5	6	V	$(16,4,4,4)$	$(1,1,1,1)$	T	global	24.66	23.91	18.06
4096	256	5	6	V	$(16,4,4,4)$	$(1,1,1,1)$	F	global	23.28	21.94	18.01
4096	256	5	6	F	$(16,4,4,4)$	$(0,0,0,0)$	T	global	25.08	24.04	15.07
4096	256	5	6	F	$(16,4,4,4)$	$(0,0,0,0)$	F	global	24.24	23.15	15.08
4096	256	6	6	V	$(4,4,4,4,4)$	$(1,1,1,1,1)$	T	local	25.82	25.36	13.52
4096	256	6	6	F	$(4,4,4,4,4)$	$(0,0,0,0,0)$	T	local	26.06	25.43	16.13
4096	256	6	6	V	$(4,4,4,4,4)$	$(1,1,1,1,1)$	T	global	30.91	29.97	13.49
4096	256	6	6	F	$(4,4,4,4,4)$	$(0,0,0,0,0)$	T	global	35.93	34.82	16.14

- Actual runtime very close to predictions
  - Same cluster based problem
- Model approaches the theoretical limit when local criterion is used
- Preceding nested iteration theoretically with zero cost

Outlook: Use task graph for scheduling

- Parareal example: $N = 6$, $K = 5$, Cost $\mathcal{G}$: 0.5 s, Cost $\mathcal{F}$: 2 s

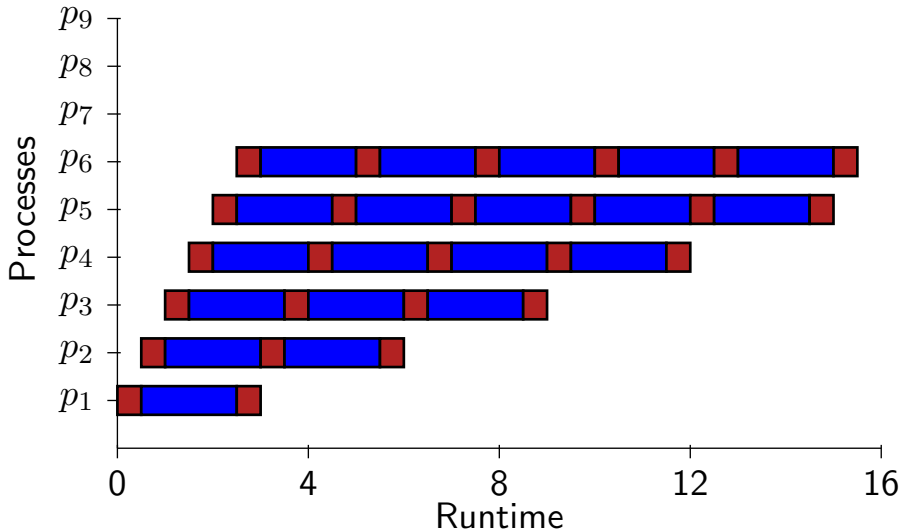


Block-by-block:

- $N_P = 6 = N$
- Runtime: 15.5 s

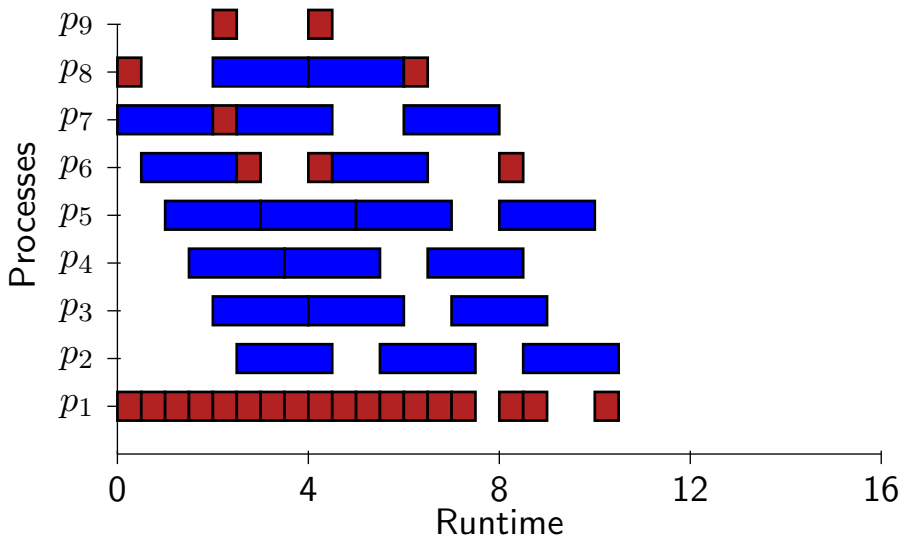
Outlook: Use task graph for scheduling

- Parareal example: $N = 6$, $K = 5$, Cost $\mathcal{G}$: 0.5 s, Cost $\mathcal{F}$: 2 s



Block-by-block:

- $N_P = 6 = N$
- Runtime: 15.5 s

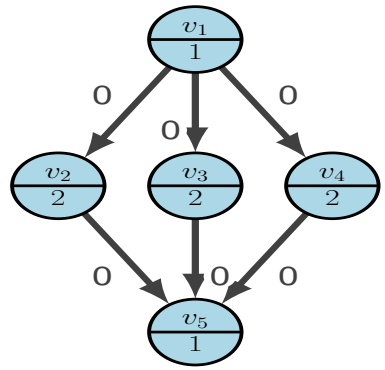


"Other" approach:

- $N_P = 9$
- Runtime: 10.5 s

Conclusions

- Task-based analysis of PinT methods



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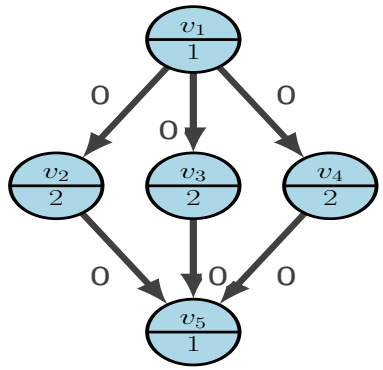
Conclusions

- Task-based analysis of PinT methods
- Covers large parameter space of PinT methods



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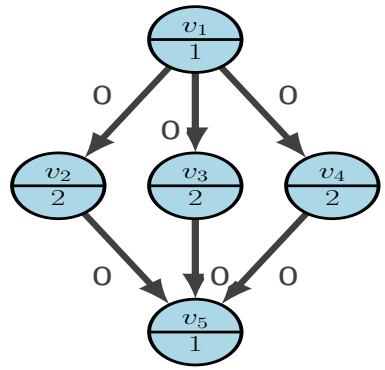
Conclusions

- Task-based analysis of PinT methods
- Covers large parameter space of PinT methods
- Theoretical lower runtime bound based on graph



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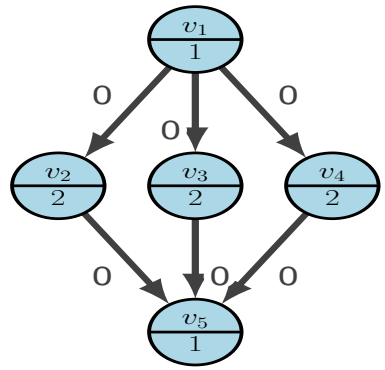
Conclusions

- Task-based analysis of PinT methods
- Covers large parameter space of PinT methods
- Theoretical lower runtime bound based on graph
- Runtime prediction using schedules



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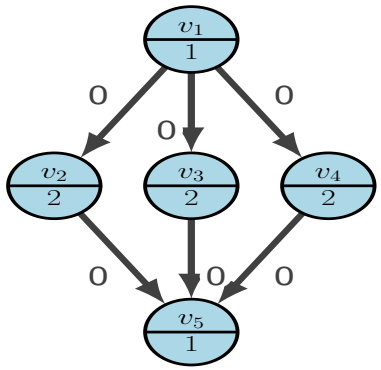
Conclusions

- Task-based analysis of PinT methods
- Covers large parameter space of PinT methods
- Theoretical lower runtime bound based on graph
- Runtime prediction using schedules
- Future work:
  - Load balancing strategies
  - Combine model with convergence predictions



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