# Poster Presentation: Bathymetry Reconstruction with Shallow Water Equations

**PinT 2022**

Judith Angel, Sebastian Götschel, Daniel Ruprecht

For the numerical simulation of tsunamis, a model of the bathymetry, the topography of the ocean bottom, is indispensable as it greatly impacts the behaviour of the wave. It is possible to approximately reconstruct the bathymetry from measurements of the water height by solving an optimisation problem with the shallow water equations as constraints. A simple approach to such PDE-constrained optimisation problems is the application of the gradient descent method to minimize the reduced objective functional. This is computationally expensive, because at each step of the iterative optimisation algorithm the governing state equations as well as backward-in-time adjoint equations have to be solved numerically to compute the gradient. In this presentation we will discuss using Parareal to speed up solving state and adjoint equations in order to reduce the time-to-solution. We will show computational results to evaluate the success of our algorithm.

# Resilience in Spectral Deferred Corrections

Thomas Baumann

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**Abstract**

Advancement in computational speed is nowadays gained by using more processing units rather than faster ones. Faults in the processing units caused by numerous sources including radiation and aging have been neglected in the past. However, the increasing size of HPC machines makes them more susceptible and it is important to develop a resilience strategy to avoid losing millions of CPU hours. Parallel-in-time methods target the very largest of computers and are hence required to come with algorithm-based fault tolerance. We look here at spectral deferred corrections (SDC), which is a time marching scheme that is at the heart of parallel-in-time methods such as PFASST. Due to its iterative nature, there is ample opportunity to plug in computationally inexpensive fault tolerance schemes, many of which are also easy to implement. We experimentally examine the capability of various strategies to recover from single bit flips both for serial SDC as well as the time-parallel extension referred to as block Gauß-Seidel SDC.

# Pint11 Poster: A Trefftz-like coarse space for the two-level Schwarz method on perforated domains

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## Abstract

We consider a new coarse space for the ASM preconditioner to solve elliptic partial differential equations on perforated domains, where the numerous polygonal perforations represent structures such as walls and buildings in urban data. With the eventual goal of modelling urban floods by means of the nonlinear Diffusive Wave equation, this contribution focuses on the solution of linear problems on perforated domains. Our coarse space uses a polygonal subdomain partitioning and is spanned by Trefftz-like basis functions that are piecewise linear on the boundary of a subdomain and harmonic inside it. It is based on nodal degrees of freedom that account for the intersection between the perforations and the subdomain boundaries. As a reference, we compare this coarse space to the well-studied Nicolaides coarse space with the same subdomain partitioning. It is known that the Nicolaides space is unable to prevent stagnation in convergence when the subdomains are not connected; we work around this issue by separating each subdomain by disconnected component. Scalability and robustness are tested on data sets based on realistic urban topography. Numerical results show that the new coarse space is very robust and accelerates the number of Krylov iterations when compared to Nicolaides, independent of the complexity of the data.

*Presenter

# A domain splitting method for the linear wave equation

Tim Buchholz [*], joint work with Pratik Kumbhar [†], Constantin Carle [‡], Marlis Hochbruck [§]

## Abstract

We construct and analyze a direct domain decomposition method for the second-order linear wave equation. This method is based on the work by Blum et al [1], in which a parabolic model problem is considered. The main idea is to use an overlapping domain decomposition to combine a cheap explicit prediction method with an implicit method for the local calculation on the subdomains. As a first step, we present the adaptation of this method to the wave equation and some numerical experiments.

## References

[1] H. Blum, and S. Lisky, and R. Rannacher, A domain splitting algorithm for parabolic problems, *Computing,*, **49** (1992), pp. 11-23.

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# Domain Decomposition Method in Time Direction for Transport Control

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## Abstract

We study the Optimized Schwarz method based on a domain decomposition in time direction (described in [2]) for control problems for the transport equation. We consider in particular internal control where the optimization problem can be transformed into a PDE system using Lagrangian (as in [1]) which unconditionally guarantee the controllability. The DD technique in time direction is then applied on this PDE system. Under Fourier analysis, we find optimal parameters for continuous and discrete cases. We illustrate the phenomena by some numerical tests.

## References

[1] M.J. Gander, F. Kwok and G. Wanner, *Constrained Optimization: From Lagrangian Mechanics to Optimal Control and PDE Constraints*, in Optimization with PDE Constraints, Springer, New York, 2014, pp. 151–202.

[2] J. E. Lagnese and G. Leugering, *Domain decomposition methods in optimal control of partial differential equations*, International Series of Numerical Mathematics, Vol. 148, Birkhäuser Verlag, Basel, 2004, pp. xiv+443

*Speaker

# Implementing the ParaDiag method using Firedrake, an automated code generation framework for the finite element method.

Josh HOPE-COLLINS (Imperial college, London, UK)

The ensemble forecasting techniques used in numerical weather prediction involve the solution of many simulations with very large sizes. This requires vast amounts of computing power, so algorithms with high scalability are essential for satisfactory performance on the massively parallel hardware of modern HPC facilities. The time- parallel ParaDiag method is one such algorithm, which exposes parallelism in the time dimension in addition to that in the spatial dimensions exposed by traditional domain-decomposition approaches. We present recent progress on the application of the ParaDiag method to finite- element discretisations of PDEs for atmospheric flow. These solvers are implemented as a general library using Firedrake, an automated code generation framework for the solution of finite-element methods. We describe our approaches to the solution of the resulting block systems and show how these can be realised within the library. The performance of these solvers is compared against theoretical convergence estimates, as well as the performance of the method for model advection problems. Various ParaDiag formulations are explored, including how the alpha-circulant approximation of the timestepping matrix is introduced, for example to approximate the Jacobian of the all-at-once system, or as a preconditioner for a Krylov subspace method. Different time-averaging procedures for the spatial Jacobian are also considered. The ParaDiag method is potentially an effective approach for finite-element discretisations of atmospheric flow models, but open questions remain on the most effective formulation for this class of problems.

**Kamran Pentlan**

**Title:** Accelerating the convergence of parareal using probabilistic methods

**Abstract:** Sequential numerical methods for integrating initial value problems (IVPs) can be prohibitively expensive when high numerical accuracy is required over the entire interval of integration. Parareal is a well-studied (deterministic) time-parallel algorithm designed to integrate such IVPs by combining solutions from cheap (coarse) and expensive (fine) numerical integrators using a predictor-corrector that locates a solution in a fixed number of iterations.

In this talk, we discuss two recently proposed algorithms, both derived using probabilistic methods, that aim to accelerate the convergence of the classic parareal algorithm.

Firstly, we introduce a "stochastic parareal" algorithm which, instead of providing the predictor-corrector with deterministically located solution values, samples and propagates M candidate values from dynamically varying probability distributions in each temporal sub-interval. For a number of simple test IVPs, we provide numerical evidence that stochastic parareal converges in fewer iterations than classic parareal, returning accurate stochastic solutions with a measure of uncertainty. Secondly, we introduce a "GParareal" algorithm which models the correction term using a Gaussian process emulator - trained on all previously collected fine and coarse solution information. Again, we demonstrate on test IVPs that GParareal converges in fewer iterations than parareal, leading to an increase in parallel speed-up. GParareal also manages to locate solutions to certain IVPs where parareal fails and has the advantageous capability of using archives of legacy solutions, e.g. solutions from prior simulations of the IVP for different initial conditions, to further accelerate convergence of the method - something that existing time-parallel methods do not do.

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All-at-once multigrid strategies for 1D space-FDEs

We focus on a time-dependent one-dimensional space-fractional diffusion equation with constant diffusion coefficients. An all-at-once rephrasing of the discretized problem yields a dense block linear system and paves the way for parallelization. In particular, in case of uniform space-time meshes, the coefficient matrix shows a two-level Toeplitz structure, and such structure can be leveraged to build ad-hoc iterative solvers that aim at ensuring an overall computational cost independent of time.

In this direction, we study the behavior of certain multigrid strategies with both semi- and full-coarsening that properly take into account the sources of anisotropy of the problem caused by the grid choice and the diffusion coefficients.

The performances of the aforementioned multigrid methods reveal sensitive to the choice of the time discretization scheme. Many tests show that Crank-Nicolson prevents the multigrid to yield good convergence results, while second-order backward-difference formula scheme is shown to be unconditionally stable and that it allows good convergence under certain conditions on the grid and the diffusion coefficients. Numerical results show the effectiveness of our proposal also in the case of variable coefficients and a 2D space.

*Joint work with* Marco Donatelli, Rolf Krause, Mariarosa Mazza.