Towards RandNLA Accelerated Parallel-in-Time Solutions

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Abstract

Full space-time discretizations of PDEs are an active research area since this approach provides opportunities to utilize many concurrent computational tasks through parallel linear algebra. Further, a full space-time discretization provides opportunities to directly apply space-time adaptivity. The challenges that arise are the resulting large (non-) linear systems that need to be solved after such a discretization. One approach to address the challenge of solving these large (non-) linear systems is to develop specialized preconditioners to accelerate the iterative solve. In this talk, we explore the construction of low-rank preconditioners using randomized NLA ideas. Suppose that the target is to solve the (large) linear system $Ax = b$. We proceed by construct a low-rank preconditioner by iteratively embedding information samples of the form

$$\Psi_k^\top A\,\Omega_k \in \mathbb{R}^{s\times s}, \quad \Psi_k \in \mathbb{R}^{m\times s}, \quad \Omega_k \in \mathbb{R}^{n\times s},$$

where $\{\Psi_k\}$ and $\{\Omega_k\}$ are so-called sampling matrices. These two-sided samples, $\Psi_k^\top A\,\Omega_k$, only have a small number of entries (s^2); consequently, our algorithm maps well to modern math co-processors which are ubiquitous in many computational platforms.