Non-overlapping Schwarz Waveform-Relaxation for Nonlinear Advection-Diffusion Equations

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Abstract

Nonlinear advection-diffusion equations often arise in the modeling of transport processes. For these equations, we propose a non-overlapping domain decomposition algorithm of Schwarz waveform-relaxation type. The method is based on nonlinear zeroth-order (or Robin) transmission conditions between the subdomains, which recover the continuity of the converged solution and of its normal flux across the interface. We prove the (unique) existence of the iterative solutions and the convergence of the algorithm. For solving the SWR problems, we present a numerical discretization using the forward Euler method in time and a finite volume method in space, including a local Newton iteration for the nonlinear transmission conditions. In the limit of vanishing diffusion, this discrete algorithm is asymptotic-preserving, and thus robust. Finally, we illustate the theoretical findings by several numerical results, particularly the robust convergence of the algorithm. Moreover, we demonstrate that the SWR algorithm can be successfully applied to two-phase porous-media flow problems as paradigms for strongly nonlinear evolution equations.

References

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