Non-overlapping domain decomposition methods for parabolic control problems

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Abstract

Non overlapping domain decomposition methods (DDMs) refer to decompositions of the domain into the union of mutually disjoint subdomains. These methods are known to be very well suited to parallel computing, and particularly efficient in many applications, for example when considering heterogeneous problems with jumps in coefficients. Since their emergence and the seminal work of Pierre-Louis Lions, they have received a considerable amount of attention. Their study, whether it is conducted at the continuous level or the discrete level, remains a challenging issue. In this study, we aim at presenting the Dirichlet-Neumann (DN) and Neumann-Neumann (NN) methods applied to optimal control problems arising from parabolic partial differential equations (PDE). This problem reads as: for a given state y governed by a parabolic PDE on the time interval $[0, T]$, we wish to drive the solution of this parabolic PDE to a desired state $\hat{y}$ through a control u. The goal into non-overlapping subdomains is to find the optimal control u^* which minimizes the discrepancy between these states (i.e. original state y and desired state $\hat{y}$). After a semi-discretization in space, we use the Lagrange multiplier approach to derive a coupled forward-backward system. This system can then be solved by using DN and NN methods by separating the time domain into two non-overlapping subdomains. Finally, we provide the convergence analysis for these two methods along with some numerical results.

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