Convergence of Schwarz Waveform Relaxation Methods for Systems of Semi-Linear Reaction-Diffusion Equations

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Abstract. Reaction-Diffusion systems play a vital role in several areas of applied mathematics. For example, SEI models are a type of Reaction-Diffusion system that is used extensively in the modelling of how diseases spread. This has been particularly relevant recently, as SEI models have been used to model the spread of COVID-19. Reaction-Diffusion systems are also used in other areas of mathematical biology, such as the Lotka-Volterra equations which are used to model predator-prey systems. Outside of mathematical biology, Reaction-Diffusion systems are used extensively in chemistry and physics. Clearly, obtaining a consistent method for solving systems of Reaction-Diffusion equations would be extremely valuable. However, in practice this proves to be very difficult. In most cases, we cannot solve the systems analytically, and must resort to using numerical algorithms to approximate a solution to the system. Additionally, Reaction- Diffusion equations often contain a non-linear term: this results in simpler methods, which are effective on linear systems, failing to converge. Therefore, we require a more sophisticated method in order to obtain solutions for these systems even with the presence of a non-linear term. Schwarz Waveform Relaxation methods, abbreviated to SWR methods, are an example of these sophisticated methods. SWR methods are a type of domain decomposition method; instead of solving the system over the entire domain on which it is defined, the domain is divided into smaller sub-domains which are solved separately. In the case of the SWR methods, these sub-domains are define such that they overlap, with transmission conditions used to transfer information between the sub-domains. The SWR methods have been studied extensively for linear partial differential equations. However, there is only a small amount of research into the convergence of the SWR methods for non-linear systems. Therefore, investigating how the methods performs on non-linear systems could prove useful in determining the effectiveness of the methods in solving Reaction-Diffusion systems in general.

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