Lagrangian controllability of the Navier-Stokes equations

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Abstract

In a joint work with Ludovick Gagnon, Toan Nguyen, and Trinh Nguyen, we address the issue of the Lagrangian controllability of the 2D Navier-Stokes equations in a bounded domain with Dirichlet "no-slip" boundary conditions. This issue was raised by Jean-Pierre Puel fifteen years ago, with the following open question: is it possible to drive a patch of particles from a place to another in finite time by the mean of an appropriate localized force term ?

In that game, one considers initial and final positions for the patches corresponding to a pair of isotopic contours, and the motion of the patch is searched with no intersection with the place where the control force is applied, which is assumed to occupy a non-empty open part of the domain. One looks for a controlled solution which reaches the targeted final position with an arbitrarily small error in an arbitrarily fine topology, within an arbitrarily given positive time.

Some earlier results have provided a positive answer in the case of the Euler equations, see [1, 2, 4], in case of the steady Stokes equations, see [3], and in the case of the Navier-Stokes equations with the Navier-slip-with-friction boundary conditions, see [5].

Another extra difficulty at stake in order to extend these investigations to the Navier-Stokes equation with the Dirichlet boundary conditions is to deal with boundary layers.

References

[1] O. Glass and T. Horsin, Approximate Lagrangian controllability for the 2-D Euler equation. Application to the control of the shape of vortex patches, *J. Math. Pures Appl. (9)*, **93** (2010), 61-90.

[2] O. Glass and T. Horsin, Prescribing the motion of a set of particles in a three-dimensional perfect fluid, *SIAM J. Control Optim.*, **50** (2012), 2726-2742.

[3] O. Glass and T. Horsin, Lagrangian controllability at low Reynolds number, *ESAIM Control Optim. Calc. Var.*, **22** (2016), 1040-1053.

[4] T. Horsin and O. Kavian, Lagrangian controllability of inviscid incompressible fluids: a constructive approach, *ESAIM Control Optim. Calc. Var.*, **23** (2017), 1179-1200.

[5] J. Liao, F. Sueur and P. Zhang, Smooth Controllability of the Navier-Stokes Equation with Navier Conditions: Application to Lagrangian Controllability. Archive for Rational Mechanics and Analysis, Issue 243, 869-941, 2022.