Quantitative control of solutions to axisymmetric Navier-Stokes equations in terms of the weak L^3 norm

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Abstract

In this joint work [3] with Stan Palasek (UCLA), we are concerned with quantitative estimates of strong axisymmetric solutions u to the 3D incompressible Navier-Stokes equations. We will discuss a recent result which shows that if $\|u\|_{L^\infty([0,T];L^{3,\infty}(\mathbb{R}^3))} \leq A$ for some $A \gg 1$ then $\|D^k u(t)\|_{L^\infty(\mathbb{R}^3)} \leq t^{-(1+k)/2} \exp\exp A^{C_k}$ for all $t \in (0,T]$, where $C_k > 0$ is a constant.

This is the first result that allows quantitative control of a solution using a norm in a critical space of weak type, which in particular includes self-similar solutions. The result can be thought of as a continuation of the work of Tao [5], who obtained control in terms of the L^3 norm, and Palasek [4], who obtained an improvement in the axisymmetric case. However the result uses a completely different approach based on Harnack inequality estimates of Nazarov and Ural'tseva [2] and energy estimates on ω_r/r and ω_θ/r developped by Chen, Fang and Zhang [1].

References

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