Dissipative solutions of compressible fluid flows: analysis and numerics

Mária Lukáčová-Medvid'ová

Institute of Mathematics, Johannes-Gutenberg University Mainz
LUKACOVA@UNI-MAINZ.DE

Abstract

In this joint work with E. Feireisl, H. Mizerová, B. She, and Y. Yuan we introduce generalized solutions of compressible flows, the so-called dissipative solutions.

We will concentrate on the inviscid flows, the Euler equations, and mention also the relevant results obtained for the viscous compressible flows, governed by the compressible Navier-Stokes equations. The existence of dissipative solutions has been shown by the convergence analysis of suitable, invariant-domain preserving finite volume schemes [1, 2, 3]. In the case that the strong solution to the above equations exists, the dissipative solutions coincide with the strong solution on its life span [1]. In this case we can also apply a novel tool of the relative entropy to derive rigorous error estimates between numerical solutions and the exact strong solution [4].

Otherwise, we apply a newly developed concept of $\mathcal{K}$-convergence and prove the strong convergence of the empirical means of numerical solutions to a dissipative solution [5, 6]. The latter is the expected value of the dissipative measure-valued solutions and satisfies a weak formulation of the Euler equations modulo the Reynolds turbulence stress tensor. In the class of dissipative solutions there exists a solution that is obtained as a vanishing viscosity limit of the Navier-Stokes system [7]. We will draw a connection to the Kolmogorov hypothesis and illustrated theoretical results by a series of numerical simulations.

The present research has been partially supported by TRR 146 Multiscale simulation methods for soft matter systems, TRR 165 Waves to Weather funded by the German Science Foundation and by the Gutenberg Research College.

References

[1] E. Feireisl, M. Lukáčová-Medvid'ová, H. Mizerová, B. She, Numerical analysis of compressible fluid flows. Springer, 2021.

[2] E. Feireisl, M. Lukáčová-Medvid'ová, H. Mizerová, Convergence of finite volume schemes for the Euler equations via dissipative-measure valued solutions, *Found. Comput. Math.* **20** (2020), 923–966.

[3] E. Feireisl, M. Lukáčová-Medvid'ová, H. Mizerová, B. She, Convergence of a finite volume scheme for the compressible Navier-Stokes system, *ESAIM: Math. Model. Num.* **53** (2019), 1957–1979.

[4] M. Lukáčová-Medvid'ová, B. She, Y. Yuan, Error estimate of the Godunov method for multidimensional compressible Euler equations, *J. Sci. Comput.* **91** (2022), 71.

[5] E. Feireisl, M. Lukáčová-Medvid'ová, H. Mizerová, $\mathcal{K}$-convergence as a new tool in numerical analysis, *IMA J. Numer. Anal.* **40** (2020), 2227–2255.

[6] E. Feireisl, M. Lukáčová-Medvid'ová, B. She, Y. Wang, Computing oscillatory solutions of the Euler system via $\mathcal{K}$-convergence, *Math. Math. Models Methods Appl. Sci.* **31** (2021), 537–576.

[7] E. Feireisl, M. Lukáčová-Medvid'ová, S. Schneider, B. She, Approximating viscosity solutions of the Euler system, *Math. Comp.* **91** (2022), 2129-2164.