

Gross-Pitaevskii at low regularity

H. Koch

Mathematical Institut, Bonn University, Germany
KOCH@MATH.UNI-BONN.DE

Abstract

In this joint work with X. Liao, we address uniform bounds for solutions to the Gross-Pitaevskii at all nonnegative regularity levels. This is based on a study of the nonlinear phase space

$$X^s = \{u \in H_{loc}^s : u_x, |u|^2 - 1 \in X^{s-1}\} / \mathbb{S}^1,$$

where we consider functions modulo the multiplication by a complex number of modulus 1. We equip this space with a metric and an analytic structure. We construct conserved Hamiltonians for all $s \geq 0$. Going below $s = \frac{1}{2}$ relies on a study of the nontrivial topology of the metric space X^s .

References

- [1] H. Koch, X. Liao, Conserved energies for the one dimensional Gross-Pitaevskii equation. *Adv. Math.* **377** (2021)
- [2] –, Conserved energies for the one dimensional Gross-Pitaevskii equation: low regularity case, *arXiv:2204.06293*, (2022).