Regularity of vortex and SQG patches

Alexander Kiselev

Department of Mathematics, Duke University
KISELEV@MATH.DUKE.EDU

Abstract

The patch solutions of the 2D Euler and (modified) SQG equations have form $\omega(x,t) = \chi_{\Omega(t)}(x)$ of a characteristic function of a domain $\Omega(t)$ evolving in time according to the Biot-Savart law $u = \nabla^{\perp}(-\Delta)^{-1+\alpha}\omega$; here $\alpha = 0$ corresponds to the Euler case and $0 < \alpha < 1$ to the modified SQG family. For the Euler case, the first proof of global regularity for pathes was given by Chemin [1] in Hölder spaces $C^{k,\beta}$, $0 < \beta < 1$. For the modified SQG family, the problem remains largely open - with the only finite time singularity formation result available in the presence of boundary and for small α [5, 2]. I will discuss some recent conditional results on the possible scenarios for finite time blow up [3]. Also, for the Euler patch case, I will describe a construction of patches that are C^2 at the initial and all integer times, but lack this regularity for all other times - without being time periodic [4]. This result is based on the analysis of the curvature evolution equation, which may also be useful for other applications.

References

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